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***State of confidence, overborrowing and macroeconomic stabilization puzzle***

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**Abstract**

In this paper we model macroeconomic instability as the outcome of the dynamical interaction between debt accumulation and the "state of confidence" in a small open economy with a super-fixed exchange rate arrangement. Our analysis is set in a theoretical framework where balance-sheets effects govern external financing to firms and the state of confidence is largely pro-cyclical. We analyse the conditions for the dominance of unstable chains in the out-of-equilibrium dynamics which determine financial fragility, systemic instability and, as a consequence, macroeconomic stabilization puzzle. Indeed, the choice of a tight fiscal policy is likely to be destabilizing inasmuch as it exacerbates the liquidity crunch taking place in the course of a recession. At the same time, a reduction in interest rates may not be sufficient to switch off macroeconomic instability, and a direct stimulus to aggregate expenditure may be required to avoid an economic collapse.

We conduct an "experimental" study with reference to Argentina during the currency board years in order to understand what the implications would have been for dynamical stability of "appropriate" monetary and fiscal policies oriented to macroeconomic stabilization. Our empirical results are based on the sensitivity analysis of a continuous-time econometric model and confirm the dangerousness of conventional austerity policies in times of recession.

**Keywords**: macrodynamical financial fragility; (in-)stability; stabilizing policy measures; sensitivity and continuous-time econometric analysis;

**Highlights**: •We present an open-economy macrodynamical model with the aim of studying financial fragility and systemic instability •We deal with an endogenous process of debt accumulation under super-fixed exchange rate arrangement •We focus on the macroeconomic policy stabilization puzzle• We perform a sensitivity analysis of the stabilizing policies on the econometric nonlinear continuous time counterpart.

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## 1. Introduction

As a stylized fact money is strongly pro-cyclical. In a small open economy, this feature mainly results from the high capital mobility induced by integration into global financial markets; the very likely outcome of liquidity endogeneity is vulnerability to systemic instability. Monetary authorities can partly neutralize this behaviour by opting for a flexible exchange rate regime. Yet, small emerging economies often choose to peg. It increases reputation regarding inflationary biases and creates expectations of sound fiscal policies. The induced monetary stability improves the "state of confidence", i.e. expectations regarding the future rate of return on capital, and creates the conditions for an acceleration of capital inflows, with beneficial effects on both private accumulation and public spending decisions. As a result, a macroeconomic expansion actually takes place, fostered by the endogenous liquidity growth. On the reverse side of the coin is the ongoing cumulative debt accumulation process, and the resulting financial fragility which occurs when agents expect the present booming profits to be maintained in the future, i.e., an endless expansion of the economy to be on the course. Indeed, it turns out that the driver of instability is exactly this combination of agents' inertia in expectations' formation, together with the presence of balance-sheet effects in credit markets, that makes the provision of finance highly dependent on the evolution of the "state of affairs". The joint association of these effects explains why an economic expansion easily turns into a fragile boost, and eventually determines a liquidity collapse.

In this paper we investigate the implications of the above features by developing a theoretical framework for the analysis of financial fragility and systemic instability in the spirit of Minsky's (1982) theoretical contribution. Our study is set in an out-of-equilibrium adjustment perspective, close to the research approach of works like Chiarella et al. (2000), Asada et al. (2010). We build on Cavallaro et al. (2011) and Maggi et al. (2012) that deal with a macro-dynamical monetary model for a small open economy, with a super-fixed exchange rate arrangement where money is completely endogenous, and dependent on the evolution of net financial inflows. There, a nonlinear real-financial interaction mechanism is at work, with liquidity accruing from abroad that stimulates investment and output, and exerts a positive impact onto the state of confidence; the latter, in turn, feeds back onto liquidity growth. In this paper we extend that analysis to the issues of stabilization policy, by investigating analytically the stabilizing as well as destabilizing effects at work in out-of-equilibrium dynamics. We show that financial fragility and systemic risk result from the combination of balance-sheet effects in credit markets and a strong inertia in agents' assessment of future profits. We show that the structure of the feedback mechanisms is crucial both for the long-run behaviour of the economy and policy effectiveness. In fact, differently from a traditional "equilibrium" framework, the adjustment dynamics we consider implies that the monetary mechanism acts on investment and output not much as in the usual way, that is, via the interest rate channel, but rather through the "state of expectations" variable, i.e., in a more Keynesian fashion. Indeed, this variable is crucial since it impacts on both the amount of finance that foreign lenders are willing to supply and firms' investments and production decisions. We show that conventional stabilization policies may not be effective, or even turn to be counter-productive: despite macroeconomic instability may result from financial fragility brought about by an over indebtedness process, the

choice of a fiscal austerity programme is likely to be destabilizing inasmuch as it exacerbates the liquidity crunch taking place in the course of a recession. At the same time, a quantitative easing activated by monetary authorities may fail in switching off instability, when the fall in interest rates does not compensate for the fall in the state of confidence, so that the "expenditure chain" in the real-financial mechanism results to be blocked.
We perform an empirical analysis relative to Argentina during the currency-board experience years. The "experimental" study we conduct is on what the implications would have been for dynamical stability of "appropriate" monetary and fiscal policies. To this aim we carry out a sensitivity analysis on stability based on the continuous-time econometric counterpart of our theoretical model. Our results confirm the theoretical analysis, thus providing a useful lesson for the Euro area countries on the dangerousness of austerity policies in a recession, and the necessity of the coordinated monetary and fiscal policies in order to stabilize a fragile economy. Our empirical approach, based on disequilibrium equations, reveals particularly suitable for its coherence with dynamical stock-flow analysis of the theory developed and for freeness from *a priori* on the existence of an equilibrium which is, possibly, an outcome.

The paper is organized as follows: section 2 develops the theoretical analysis of the long run dynamical behaviour of the economy, section 3 the economic policy implications, section 4 the empirical analysis, section 5 concludes.

## 2. The theoretical setup

### 2.1 The model

In our model there are two there are two type of agents, workers that consume entirely their income, and capitalists that save and make investment decisions. For the sake of simplicity, we consider a fixed-price setting.[1] Financing to firms accrues exclusively from foreign lending. The private sector is characterized –as follows. All variables are expressed in units of capital. Private saving is the share of profit, $\alpha$, out of income, $y$, net of interest payment, $il_p$, $k \equiv I/K$ is the rate of investment on the stock of capital

[2.1] $$k = \gamma\left[(\pi_n^e - i\right], \qquad \gamma' > 0,$$

where $\pi_n^e \equiv \pi + \rho - il_p$ is the expected net profit rate, defined as the current profit rate, $\pi$, plus its expected change, $\rho$, net of interest payments on firms' outstanding debt. Interest payments to foreign lenders on outstanding debt $l_p$ are assumed to be instantaneous. The above formulation states that investment decisions are made on the basis of the expected relative profitability of investment in new capital goods, which is a function of the difference between the marginal efficiency of capital and its (replacement) cost, in the spirit of the Keynesian tradition.

[1] The assumption of fixed prices is made in order to allow analytical tractability of the dynamical system. Removing the assumption would not affect the results of our analysis.

As to the public sector, collected taxes are in part autonomous, in part proportional to income, $\tau = (\tau_0 + \tau_1 y)$, with $\tau_0 > 0$, $0 < \tau_1 < 1$, and public spending has a discretional component dependent on the cyclical behaviour of the economy, $g = (\beta_0 + \beta_1 \rho)$, with $\beta_0 > 0$, $\beta_1 < |1|$. The government's net saving $s_g$ is thus the difference between collected taxes and government's purchase of goods and services and the reimburse of interests $il_g$, that is

[2.2] $$s_g = (\tau_0 + \tau_1 y) - (\beta_0 + \beta_1 \rho) - il_g .$$

This specification implies a constant ratio $g \equiv G/K$ for $\beta_1 = 0$ or that the government runs a pro-cyclical ($\beta_1 > 0$) or anti-cyclical ($\beta_1 < 0$) public deficit spending. The specific policy depends, of course, on the government's preferences and commitments.

As to the foreign sector, we consider net income from abroad as resulting from the current account

[2.3] $$ca = nx - il \quad , \qquad l = l_p + l_g$$

where *nx* is net exports, and *il* overall interest payments to non residents. Regarding net exports, as standard we posit a positive relationship with domestic income, and include a exogenous terms that account for foreign income ($y_w$), and the real exchange rate ($x$), that is

[2.4] $$nx = n_1 y - n_2 y_w - n_3 x \text{, with } n_1 < 0,\ n_2 > 0, n_3 > 0$$

The monetary side of the model is strongly dependent on the exchange rate arrangement. On the assumption that the country adopts a super-fixed exchange rate, i.e., a currency board, or a dollarization (euroization), at each time *t* the stock of money ought to be equal, or proportional, to the amount of foreign currency in the economy. This makes money completely endogenous and dependent on the evolution of the balance of payments. With no loss of generality, we assume $r_{t_0} = m_{t_0}$ , so that at each time $t$ the stock of reserves, and thus, the stock of money are obtained by integration of the balance of payments identity, $\dot{r}_t = \dot{l} - ca$, where $r$ is foreign reserves and $\dot{l}$ financial inflows. We thus get

[2.5] $$m_t = r_t = l_t - \int_{v=0}^{t} il_v \, dv + \int_{v=0}^{t} nx_v \, dv ,$$

As to the demand for money, we consider the conventional function with the level of income and the interest rate as arguments

[2.6] $$m^d = m^d(y,i) \qquad m_1^d > 0, \quad m_2^d < 0 \qquad .$$

As common in stock-flow analyses we distinguish between the short-run and the long-run behaviour of the model. We posit that, in the short run, equilibrium in the goods and money markets is achieved instantaneously, for given values of the variables that change through time, in particular, money, foreign reserves, the state of confidence.[2] The model is then closed analytically by positing the laws of motion through time of the above variables, thus providing the long run dynamical behaviour of the system. Given the functional forms [2.1]-[2.6], the following equilibrium conditions for the goods and money markets -being *s* the total aggregate saving- are supposed to hold over time:

[2.7] $$s - k - ca = 0$$

[2.8] $$m - m^d = 0$$

Equations [2.7] and [2.8] jointly determine the level of output and the interest rate that, at each time *t*, ensure equilibrium in the goods and money markets, respectively, for given (temporary-equilibrium) values of $l_p, l_g, \rho$, that is, $y = \Psi(l_p, l_g, \rho)$ and $i = \Theta(l_p, l_g, \rho)$, respectively. The dynamical characterization of the model is then obtained by adding the laws of motion of the above variables, as follows.

As to the stock of private debt per unit of capital, $l_p$, we can express its time derivative as $\dot{l}_p = (\varphi(\cdot) - k)l_p$ where $\varphi(\cdot)$ is a function that explains the accumulation of private debt through foreign loans. In this respect, we assume that financing to firms accrues exclusively from foreign lending, and that the existence of information problems makes it difficult for lenders to assess the probability of loans' repayment on the basis of standard price mechanisms. We thus posit that foreign lenders use very simple rules to assess firms' worthiness, in particular they look at two indicators: the expected net rate of return on investment, $\pi^e - i$, and firms' degree of leverage, $l_p$. Whereas the former variable captures the "economic" value of the project to be financed, the latter provides information on firms' financial situation, and therefore on the financial risk incurred by lenders. Financing to firms is thus supply-side driven on the basis of the following law of accumulation of private debt through time[3]

[2.9] $$\dot{l}_p = \left[\varphi\left(\pi^e - i; l_p\right) - k\left(\pi^e - i\right)\right] l_p \qquad \varphi_1 > 0,\ \varphi_2 < 0$$

As to the government's debt, we assume that public institutions face no constraints in financing deficit spending through the issue of debt, that is, the pricing of risk is not a concern in the case of public debt. This amounts to assuming that relevant information to creditors is provided by international rating agencies, and reputational concerns induce governments to commit themselves to the repayment of debt obligations, so that adverse incentives problems are not at issue. The accumulation of public debt through time is thus demand-determined, and equal to the government's budget deficits over time. Recalling that variables are in units of capital, we have

[2.10] $$\dot{l}_g = -s_g + il_g - kl_g \ .$$

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[2]The assumption of an instantaneous adjustment in these two markets is here made for the analytical tractability of the model, and will be removed in the empirical analysis.

[3] The function $\varphi(.)$ thus provides the macro-dynamical representation of balance-sheet effects in credit markets, as suggested in Franke and Semmler (1989).

As to $\rho$, it is supposed to capture the economy's "state of confidence", in a Minskian perspective. Its evolution through time provides a representation of the way firms, as well as lenders, assess the evolution of profitability conditions of investment projects. Because of the limited information-set available, such an assessment is made on the basis of a simple adaptive rule.[4] Yet, when information is incomplete or asymmetric, the value of investment projects is not independent from firms' financial structure, and higher levels of debt signal an increase in the probability of bankruptcy. We thus assume that, in forming expectations regarding future profitability of investments, agents behave adaptively by looking at the *current* profitability, $\pi - i$, as an indicator of the economic return to investment, but then adjust their assessment on the basis of the degree of firms' degree of leverage, which provides a measure of the private sector "financial" robustness, and on the public sector's degree of leverage $l_g$. The idea is that what matters for systemic stability is the overall debt accumulated over time. We thus posit:

[2.11] $$\dot{\rho} = \omega(\pi - i; l_p; l_g) \quad \omega_1 > 0,\ \omega_2 < 0,\ \omega_3 < 0$$

By adding the laws of motion [2.9] – [2.11] to the goods and money markets equilibrium conditions given in above equations [2.7] – [2.8], we obtain the following system [S.1] of our model:

[S.1] $$\begin{cases} \alpha y - \gamma\left[\left(\alpha y + \rho - il_p\right) - i\right] - \beta_1 \rho + \tau_1 y + C = 0 \\ l_p + l_g + \int ca(y,i) - m^d(y,i) = 0 \\ \dot{l}_p = \left[\phi\left(\left(\alpha y + \rho - il_p\right) - i;\, l_p\right) - \gamma\left(\left(\alpha y + \rho - il_p\right) - i\right)\right] l_p \\ \dot{l}_g = (\beta_0 - \tau_0) + \beta_1 \rho - \tau_1 y + il_g - \gamma\left(\left(\alpha y + \rho - il_p\right) - i\right) l_g \\ \dot{\rho} = \omega\left(\left(\alpha y - il\right) - i; l_p; l_g\right) \end{cases}$$

Where $C = \tau_0 - \beta_0 - n_2 y_w - n_3 x$ , in the first equation, denotes the constants.

The links between the domestic money market, interest-rate spread and foreign reserves are the result of the peculiar currency exchange rate arrangement. First, given the currency board arrangement, the economy's overall liquidity is determined by the stock of reserves cumulated through the balance of payment net inflows, as composed by the current account and financial flows to the private and public sectors. These flows are represented by the laws of motion given in equations [2.9] and [2.10], respectively[5]. Second, the interest rate in the domestic money market is determined in equation [2.8], on the basis of the overall liquidity created endogenously in the economy, and its evolution through time depends in particular on the occurrences in the private sector, which determine the strength of the balance-sheet effects in the

[4] The role of heuristics in the presence of limited cognitive skills is investigated in Brock and Hommes (1997). More recent applications of heuristics to financial markets are in Lux and Marchesi (2000), De Grauwe and Grimaldi (2006), De Grauwe (2009).

[5] We assume residents purchases of foreign financial assets to be negligible.

credit market (eq. 2.9): in the boom, foreign lenders accommodate easily firms demand for finance, whereas the opposite happens during a contraction. The above mechanism determines the abundance or scarcity of liquidity in the economy and – given the international interest rate - the magnitude of the risk premium over the cycle.

## 2.2 Convergence and long-run dynamics

We now study the long-run dynamical behaviour of the economy at the steady-state equilibrium. We first obtain the 3D fundamental dynamical system for [S.1] by substitution of the temporary-equilibrium solutions of output and the interest rate $y=\Psi(l_p,l_g,\rho)$ and $i=\Theta(l_p,l_g,\rho)$ that solve the first two equations, in the subsequent three laws of motion. We then posit that, in the steady state, profitability expectations and both stocks of debt per unit of capital are unchanging through time. Accordingly, output per unit of capital and the interest rate are constant, too. The following conditions are then satisfied:

[2.12]

$$\dot{l}_p=\left\{\varphi\left[\alpha\,\Psi\left(l_p^*,l_g^*,\rho^*\right)-\Theta\left(l_p^*,l_g^*,\rho^*\right)\left(1+l_p^*\right)+\rho^*;\,l_p^*\right]-\gamma\left[\alpha\,\Psi\left(l_p^*,l_g^*,\rho^*\right)-\Theta\left(l_p^*,l_g^*,\rho^*\right)\left(1+l_p^*\right)+\rho^*\right]\right\}l_p^*\equiv F^{\dot{l}_p}\left(l_p^*,l_g^*,\rho^*\right)=0$$

[2.13]

$$\dot{l}_g=\left(\beta_0-\tau_0\right)+\beta_1\rho^*+\Theta\left(l_p^*,l_g^*,\rho^*\right)l_g^*-\tau_1\,\Psi\left(l_p^*,l_g^*,\rho^*\right)-\gamma\left[\alpha\,\Psi\left(l_p^*,l_g^*,\rho^*\right)-\Theta\left(l_p^*,l_g^*,\rho^*\right)\left(1+l_p^*\right)+\rho^*\right]l_g^*\equiv F^{\dot{l}_g}\left(l_p^*,l_g^*,\rho^*\right)=0$$

[2.14] $$\dot{\rho}=\omega\left[\alpha\,\Psi\left(l_p^*,l_g^*,\rho^*\right)-\Theta\left(l_p^*,l_g^*,\rho^*\right)\left(1+l_p^*\right);\,l_p^*;l_g^*\right]\equiv F^{\dot{\rho}}\left(l_p^*,l_g^*,\rho^*\right)=0$$

where $l_p^*$, $l_g^*$ and $\rho^*$ denote the steady-state values of the stocks of private debt, public debt, and the state of confidence, respectively, and where $y=\Psi(l_p^*,l_g^*,\rho^*)$ and $i=\Theta(l_p^*,l_g^*,\rho^*)$ the steady-state values of output and the interest rate.

The local stability analysis of the system is studied by evaluating the Jacobian matrix of the fundamental dynamical system

[2.15] $$J=\begin{bmatrix} F_{l_p}^{\dot{l}_p^*} & F_{l_g}^{\dot{l}_p^*} & F_{\rho}^{\dot{l}_p^*} \\ F_{l_p}^{\dot{l}_g^*} & F_{l_g}^{\dot{l}_g^*} & F_{\rho}^{\dot{l}_g^*} \\ F_{l_p}^{\dot{\rho}^*} & F_{l_g}^{\dot{\rho}^*} & F_{\rho}^{\dot{\rho}^*} \end{bmatrix},$$

where the elements of matrix $J$ are the partial derivatives of the functions $F^{\dot{l}_p}$, $F^{\dot{l}_g}$ and $F^{\dot{\rho}}$, given in equations [2.11]-[2.14], evaluated around the equilibrium point, that is:

*(i)* $$F_{l_p}^{\dot{l}_p^*}\equiv F_{11}=\left\{\left(\varphi_1^*-\gamma'^*\right)\left[\alpha\,\Psi_{l_p}^*-\Theta_{l_p}^*\left(1+l_p^*\right)-\Theta^*\right]+\varphi_2^*\right\}l_p^*$$

$$(ii) \quad F_{l_g}^{\dot{l}_p^*} \equiv F_{12} = \left(\varphi_1^* - \gamma'^*\right)\left[\alpha\, \Psi_{l_g}^* - \Theta_{l_g}^*\left(1 + l_p^*\right)\right] l_p^*$$

$$(iii) \quad F_{\rho}^{\dot{l}_p^*} \equiv F_{13} = \left(\varphi_1^* - \gamma'^*\right)\left[\alpha\, \Psi_{\rho}^* - \Theta_{\rho}^*\left(1 + l^*\right) + 1\right] l_p^*$$

$$(iv) \quad F_{l_p}^{\dot{l}_g^*} \equiv F_{21} = \Theta_{l_p} l_g - \tau_1 \Psi_{l_p}^* - \gamma'^* \left[\alpha\, \Psi_{l_p}^* - \Theta_{l_p}^*\left(1 + l_p^*\right) - \Theta^*\right] l_g^*$$

$$(v) \quad F_{l_g}^{\dot{l}_g^*} \equiv F_{22} = \Theta_{l_g} l_g + \Theta - \tau_1 \Psi_{l_g}^* - \gamma'^* \left[\alpha\, \Psi_{l_g}^* - \Theta_{l_g}^*\left(1 + l_p^*\right)\right] l_g^*$$

$$(vi) \quad F_{\rho}^{\dot{l}_g^*} \equiv F_{23} = \beta_1 + \Theta_{\rho} l_g - \tau_1 \Psi_{\rho}^* - \gamma'^* \left[\alpha\, \Psi_{\rho}^* - \Theta_{\rho}^*\left(1 + l_p^*\right) + 1\right] l_g^*$$

$$(vii) \quad F_{l_p}^{\dot{\rho}^*} \equiv F_{31} = \omega_1 \left[\alpha\, \Psi_{l_p}^* - \Theta_{l_p}^*\left(1 + l_p^*\right) - \Theta^*\right] + \omega_2^*$$

$$(viii) \quad F_{l_g}^{\dot{\rho}^*} \equiv F_{32} = \omega_1^* \left[\alpha\, \Psi_{l_g}^* - \Theta_{l_g}^*\left(1 + l_p^*\right)\right] + \omega_3^*$$

$$(ix) \quad F_{\rho}^{\dot{\rho}^*} \equiv F_{33} = \omega_1^* \left[\alpha\, \Psi_{\rho}^* - \Theta_{\rho}^*\left(1 + l_p^*\right)\right]$$

By looking at the dynamical system [2.12] - [2.14], it appears that the equilibrium levels $l_p^*$, $l_g^*$ and $\rho^*$ impact on the law of motion of $l_p$, $l_g$ and $\rho$ directly, but also indirectly through an effect onto the temporary equilibrium values of the profit rate and the interest rate. In particular, the partial derivatives $\gamma' > 0$, $\varphi_1 > 0$, $\varphi_2 < 0$ and $\omega_1 > 0$, $\omega_2 < 0$ and $\omega_3 < 0$ capture the direct effects, whereas $\Psi_{l_p}^* > 0$, $\Psi_{l_g}^* > 0$, $\Psi_{\rho}^* > 0$, $\Theta_{l_p}^* < 0$, $\Theta_{l_g}^* < 0$, $\Theta_{\rho}^* > 0$ the indirect effects. Overall, the signs of the elements in the Jacobian matrix depend on the combination of the above direct and indirect effects. For instance in (*i*), an increase in $l_p$ exerts indirectly a positive effect onto liquidity growth, i.e., the flow of finance, through the impact on the net profit rate, measured by the derivatives in the square brackets, and a negative direct leverage effect measured by $|\varphi_2|$. The overall effect is negative, i.e., $F_{l_p}^{\dot{l}_p^*} < 0$, if lenders' sensitivity to firms' degree of leverage is strong and dominates, given that $\varphi_1 - \gamma' > 0$. Analogously, in equations (*vii*) and (*viii*), $F_{l_p}^{\dot{\rho}^*} < 0$ and $F_{l_g}^{\dot{\rho}^*} < 0$ if $|\omega_2^*|$ and $|\omega_3^*|$ are large enough, so that the direct (negative) impact of an increase in private and public leverage onto the state of confidence change offsets the positive liquidity effect operating indirectly onto net profitability - the term in square brackets. This follows from the twofold nature of finance: it provides the means for capital accumulation and economic growth, but at the same time leads to debt accumulation and balance sheets' deterioration; the net effect on the state of confidence depends on the relative magnitude of the coefficients, i.e., on lenders' sensitivity to financial fragility.

Overall, we get the following signs for the partial derivatives in equations (*i*)-(*ix*):

$F_{l_p}^{\dot{l}_p^*} \gtrless 0$; $F_{l_g}^{\dot{l}_p^*} > 0$; $F_{\rho}^{\dot{l}_p^*} > 0$; $F_{l_p}^{\dot{l}_g^*} < 0$; $F_{l_g}^{\dot{l}_g^*} < 0$; $F_{\rho}^{\dot{l}_g^*} \gtrless 0$ ; $F_{l_p}^{\dot{\rho}^*} \gtrless 0$ ; $F_{l_g}^{\dot{\rho}^*} \gtrless 0$ ; $F_{\rho}^{\dot{\rho}^*} \gtrless 0$.

In order to assess the stability properties of the above system we can resort to Routh theorem on the convergence of a time path. The theorem states that the real part of all the roots of an *n*-th degree polynomial equation are negative if and only if the sequence of determinants built on the odd and even coefficients, numbered according to the degree of the polynomial, are all positive[6], and that this is true if, defining the characteristic polynomial as $a_0\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$, with $a_0 = 1$, it happens that

[2.16]

$$a_1 = -tr\left|J\right| = -\left[F_{l_p}^{\dot{l}_p^*} + F_{l_g}^{\dot{l}_g^*} + F_{\rho}^{\dot{\rho}^*}\right] > 0$$

[2.17]

$$a_2 = \sum prin.\min\det\left|J\right| = F_{l_g}^{\dot{l}_g^*}F_{\rho}^{\dot{\rho}^*} - F_{l_g}^{\dot{\rho}^*}F_{\rho}^{\dot{l}_g^*} + F_{l_p}^{\dot{l}_p^*}F_{\rho}^{\dot{\rho}^*} - F_{l_p}^{\dot{\rho}^*}F_{\rho}^{\dot{l}_p^*} + F_{l_p}^{\dot{l}_p^*}F_{l_g}^{\dot{l}_g^*} - F_{l_p}^{\dot{l}_g^*}F_{l_g}^{\dot{l}_p^*} > 0$$

[2.18]

$$a_3 = -\det\left|J\right| =$$

$$-\left[F_{l_p}^{\dot{l}_p^*}F_{l_g}^{\dot{l}_g^*}F_{\rho}^{\dot{\rho}^*} - F_{l_p}^{\dot{l}_p^*}F_{\rho}^{\dot{l}_g^*}F_{l_g}^{\dot{\rho}^*} + F_{l_g}^{\dot{l}_p^*}F_{\rho}^{\dot{l}_g^*}F_{l_p}^{\dot{\rho}^*} - F_{l_g}^{\dot{l}_p^*}F_{l_p}^{\dot{l}_g^*}F_{\rho}^{\dot{\rho}^*} + F_{\rho}^{\dot{l}_p^*}F_{l_p}^{\dot{l}_g^*}F_{l_g}^{\dot{\rho}^*} - F_{\rho}^{\dot{l}_p^*}F_{l_g}^{\dot{l}_g^*}F_{l_p}^{\dot{\rho}^*}\right] > 0$$

and finally the product between [2.16] and [2.17] minus [2.18] is positive, i.e.

[2.19]

$$-F_{l_p}^{\dot{l}_p^*}F_{l_p}^{\dot{l}_p^*}F_{\rho}^{\dot{\rho}^*} + F_{l_p}^{\dot{l}_p^*}F_{l_p}^{\dot{\rho}^*}F_{\rho}^{\dot{l}_p^*} - F_{l_p}^{\dot{l}_p^*}F_{l_p}^{\dot{l}_p^*}F_{l_g}^{\dot{l}_g^*} + F_{l_p}^{\dot{l}_p^*}F_{l_p}^{\dot{l}_g^*}F_{l_g}^{\dot{l}_p^*} - F_{l_g}^{\dot{l}_g^*}F_{l_g}^{\dot{l}_g^*}F_{\rho}^{\dot{\rho}^*} + F_{l_g}^{\dot{l}_g^*}F_{l_g}^{\dot{\rho}^*}F_{\rho}^{\dot{l}_g^*} - F_{l_g}^{\dot{l}_g^*}F_{l_p}^{\dot{l}_p^*}F_{\rho}^{\dot{\rho}^*} +$$

$$-F_{l_g}^{\dot{l}_g^*}F_{l_p}^{\dot{l}_p^*}F_{l_g}^{\dot{l}_g^*} + F_{l_g}^{\dot{l}_g^*}F_{l_p}^{\dot{l}_g^*}F_{l_g}^{\dot{l}_p^*} - F_{\rho}^{\dot{\rho}^*}F_{l_g}^{\dot{l}_g^*}F_{\rho}^{\dot{\rho}^*} + F_{\rho}^{\dot{\rho}^*}F_{l_g}^{\dot{\rho}^*}F_{\rho}^{\dot{l}_g^*} - F_{\rho}^{\dot{\rho}^*}F_{l_p}^{\dot{l}_p^*}F_{\rho}^{\dot{\rho}^*} + F_{\rho}^{\dot{\rho}^*}F_{l_p}^{\dot{\rho}^*}F_{\rho}^{\dot{l}_p^*} - F_{\rho}^{\dot{\rho}^*}F_{l_p}^{\dot{l}_p^*}F_{l_g}^{\dot{l}_g^*}$$

$$+F_{l_g}^{\dot{l}_p^*}F_{\rho}^{\dot{l}_g^*}F_{l_p}^{\dot{\rho}^*} + F_{\rho}^{\dot{l}_p^*}F_{l_p}^{\dot{l}_g^*}F_{l_g}^{\dot{\rho}^*} > 0$$

It can be checked that condition [2.16] is satisfied for a relatively strong sensitivity of lenders to firms' leverage – a high value of $\left|\varphi_2\right|$, that ensures $F_{l_p}^{\dot{l}_p^*} < 0$, coupled with a negative weak indirect impact of expectations onto the temporary-equilibrium value of the net profit rate, $\alpha y - i$, i.e., $F_{\rho}^{\dot{\rho}^*} < 0$. In the case $F_{\rho}^{\dot{\rho}^*} > 0$ , its value should be small enough to ensure the negativity of the trace.

---

[6] For instance, for the polynomial $a_0\lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + ... + a_{n-1}\lambda^{n-1} + a_n\lambda^n = 0$, the sequence of determinants will be $\left|a_1\right| > 0$, $\begin{vmatrix} a_1 & a_3 \\ a_0 & a_2 \end{vmatrix} > 0$, $\begin{vmatrix} a_1 & a_3 & a_5 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix} > 0$, $\begin{vmatrix} a_1 & a_3 & a_5 & a_7 \\ a_0 & a_2 & a_4 & a_6 \\ 0 & a_1 & a_3 & a_5 \\ 0 & a_0 & a_2 & a_4 \end{vmatrix} > 0$, .....(See Gandolfo 2010).

As to conditions [2.17]-[2.19], the resulting effect of stabilizing and destabilizing forces at work in the nonlinear interaction determines whether they are satisfied or not. In particular, it may be checked that, throughout the above conditions, stability requires the joint products $F_{l_p}^{\dot{\rho}^*} F_{\rho}^{\dot{l}_p^*}$ and $F_{l_g}^{\dot{\rho}^*} F_{\rho}^{\dot{l}_g^*}$ to be negative. When these joint effects are positive, the interaction between the evolution of expectations and liquidity growth determines a self-enhancing destabilizing process. It follows that, for self-sustaining destabilizing oscillations to be ruled out, in eq. *(vii)* we need to restrict $F_{l_p}^{\dot{\rho}^*}$ to $F_{l_p}^{\dot{\rho}^*} < 0$, since in eq. *(iii)* $F_{\rho}^{\dot{l}_p^*} > 0$. Economically, this amounts to assuming that leverage considerations matter and dominate liquidity considerations in eq. *(viii).* With the same reasoning, a low sensitivity of expectations to government's debt which made $F_{l_g}^{\dot{\rho}^*} \rangle 0$ in eq. *(viii)* would require an anti-cyclical fiscal policy, $\beta_1$<0 in eq, (*vi*), to ensure $F_{\rho}^{\dot{l}_g^*} < 0$. Overall, stability hinges on the prevalence of stabilizing feed-back mechanisms ensured by a high sensitivity of lenders to the amount of debt being accumulated in the private sector, and a sufficient high sensitivity of expectations to the private and public sectors' leverage. Yet, the feed-back effects at work might well be destabilizing, and the model display an out-of-equilibrium dynamics.

In conclusion, the model may exhibit different behaviours, depending on the assumptions regarding the various reaction functions, which basically reflect the attitude of the economy to incur into external overborrowing. This may happen in periods of expanding economic activity when firms' expectations of blooming future profits speed up investment activity, and expectations of persistent output growth increase the governments' deficit spending. If foreign lenders easily accommodate the increase in the demand for finance, the building up of debt in firms' and government's balance sheets may lead to financial fragility. When increasing interest rates reduce profitability conditions, and lenders become unwilling to provide new finance, a downward swing may start. The loss of confidence in the currency arrangement may then exacerbate the perception of financial fragility, and the lower levels of debt may not be sufficient to restore profitability expectations and restart business activity. The economy may thus be exposed to a capital flight. Since in the model money is completely endogenous, a financial collapse may cause macroeconomic instability, with no other way out than abandoning the currency arrangement.

## 3. Financial fragility, systemic instability and the macroeconomic stabilization puzzle

Once instability is recognized as the endogenous outcome of the dynamical behaviour of any economy based on debt accumulation, the issue of detecting the proper policies capable of stabilizing an unstable economy is of outmost importance. From the analysis developed it is clear that a given policy will result effective if and only if it actually impacts on the feedback mechanisms that propel instability. Indeed, given the macrodynamical representation of the behaviour of the system, a mix of policies is more likely to be effective in controlling the instability stemming from the unstable dynamical chains of the model. In fact, despite macroeconomic instability may be the result o an over-indebtedness process, a deleveraging brought about by a tight fiscal policy is likely to be destabilizing, inasmuch as it exacerbates the liquidity problems of the economy. The fall in government spending adds to the cut in private spending in course, thus amplifying the fall in output, and the deterioration of the state of confidence.

In terms of the dynamical analysis above, a tightening in fiscal policy, indicated by a reduction in the parameter that measures the fiscal policy stance, $\beta_1$, would have no effect in eliminating the economy's instability, when this were due, for instance, to an excessively low value of lenders' sensitivity to firms' leverage, $\varphi_2$, which made $F_{l_p}^{i_p^*} > 0$ in eq. *(i).* Analytically, this case would be represented by the fail of condition [2.16], i.e., a negative trace of the Jacobian matrix, that would not be restored by a change in the fiscal policy parameter.

Would an autonomous monetary policy prevent the liquidity collapse by means of a prompt liquidity fuel? The answer depends on the monetary mechanism ability to activate the expenditure channel, so that the demand stimulus from the interest rate reduction compensates for the fall in the state of confidence.

The above considerations are in line with Minsky's recommendations: what is needed to prevent systemic instability is a "big State" and a "big Bank", that is, monetary policy *per se* can result ineffective when the state of confidence-liquidity mechanism that governs the system is like the one represented (see Minsky 1982, 1986).

To deal with the above issues analytically, suppose the monetary authorities deviate from the super-fixed exchange rate commitment, so as to keep the stock of money under control. Assume a simple anti-cyclical rule, such that liquidity is increased whenever it is scarce, with respect to a "normal", or average, level. With no loss of generality, we may suppose such scarcity to be signalled by the deviation of the interest rate from its "normal" or average level. The equations of the monetary side of the model are now modified as

[3.1] $m_t = h_t + r_v$

where *h* is the "controlled" component of money, in units of capital, with

[3.2] $\dot{h} = \left[ v(i - \bar{i}) - k \right] h$

where $v(.)$ is the monetary authorities reaction function.

Taking into account equations [3.1] and [3.2], the dynamical system [S.1] turns to the following:

$$
[S.2]\begin{cases}
\alpha y-\gamma\left[\left(\left(\alpha y+\rho-il_p\right)-i\right]-\beta_1\rho+\tau_1 y+C=0\right.\\
l_p+l_g+h+\int ca(y,i)-m^d(y,i)=0\\
\dot{l}_p=\left[\varphi\left(\left(\alpha y+\rho-il_p\right)-i;\ l_p\right)-\gamma\left(\left(\alpha y+\rho-il_p\right)-i\right)\right]l_p\\
\dot{l}_g=\left(\beta_0-\tau_0\right)+\beta_1\rho-\tau_1 y+il_g-\gamma\left(\left(\alpha y+\rho-il_p\right)-i\right)l_g\\
\dot{\rho}=\omega\left(\left(\alpha y-il\right)-i;l_p;l_g\right)\\
\dot{h}=\left[v(i-\bar{i})-\gamma\left(\left(\alpha y+\rho-il_p\right)-i\right)\right]h
\end{cases}
$$

The temporary-equilibrium values of output and the interest rate that solve the first two equations are now also function of the stock of money controlled by the monetary authorities, *h*, that is, $y=\Psi(l_p,l_g,\rho\ ,h)$ and $i=\Theta(l_p,l_g,\rho\ ,h)$. By substitution of these values into the four laws of motion of system [S.2] we can write the following conditions for the 4D fundamental dynamical system, which need to be satisfied at the steady state:

[3.3]

$$
\dot{l}_p=\left\{\varphi\left[\alpha\,\Psi\left(l_p^*,l_g^*,\rho^*,h^*\right)-\Theta\left(l_p^*,l_g^*,\rho^*,h^*\right)\left(1+l_p^*\right)+\rho^*;\ l_p^*\right]-\gamma\left[\alpha\,\Psi\left(l_p^*,l_g^*,\rho^*,h^*\right)-\Theta\left(l_p^*,l_g^*,\rho^*,h^*\right)\left(1+l_p^*\right)+\rho^*\right]\right\}l_p^*\equiv
$$
$$
F^{\dot{l}_p}\left(l_p^*,l_g^*,\rho^*,h^*\right)=0
$$

[3.4]

$$
\dot{l}_g=\left(\beta_0-\tau_0\right)+\beta_1\rho^*+\Theta\left(l_p^*,l_g^*,\rho^*,h^*\right)l_g^*-\tau_1\,\Psi\left(l_p^*,l_g^*,\rho^*,h^*\right)-\gamma\left[\alpha\,\Psi\left(l_p^*,l_g^*,\rho^*,h^*\right)-\Theta\left(l_p^*,l_g^*,\rho^*,h^*\right)\left(1+l_p^*\right)+\rho^*\right]l_g^*\equiv
$$
$$
F^{\dot{l}_g}\left(l_p^*,l_g^*,\rho^*,h^*\right)=0
$$

[3.5] $$\dot{\rho}=\omega\left[\alpha\,\Psi\left(l_p^*,l_g^*,\rho^*,h^*\right)-\Theta\left(l_p^*,l_g^*,\rho^*,h^*\right)\left(1+l_p^*\right);\ l_p^*;l_g^*\right]\equiv F^{\dot{\rho}}\left(l_p^*,l_g^*,\rho^*,h^*\right)=0$$

[3.6]

$$
\dot{h}=\left\{v\left[\Theta\left(l_p^*,l_g^*,\rho^*,h^*\right)-\bar{i}\ \right]-\gamma\left[\alpha\,\Psi\left(l_p^*,l_g^*,\rho^*,h^*\right)-\Theta\left(l_p^*,l_g^*,\rho^*,h^*\right)\left(1+l_p^*\right)+\rho^*\right]\right\}h^*\equiv
$$
$$
F^{\dot{h}}\left(l_p^*,l_g^*,\rho^*,h^*\right)=0
$$

The Jacobian of the 4D dynamical system [3.3]-[3.6] is:

[3.7] $$J^H=\begin{bmatrix}
F_{l_p}^{\dot{l}_p^*} & F_{l_g}^{\dot{l}_p^*} & F_{\rho}^{\dot{l}_p^*} & F_{h}^{\dot{l}_p^*}\\
F_{l_p}^{\dot{l}_g^*} & F_{l_g}^{\dot{l}_g^*} & F_{\rho}^{\dot{l}_g^*} & F_{h}^{\dot{l}_g^*}\\
F_{l_p}^{\dot{\rho}^*} & F_{l_g}^{\dot{\rho}^*} & F_{\rho}^{\dot{\rho}^*} & F_{h}^{\dot{\rho}^*}\\
F_{l_p}^{\dot{h}^*} & F_{l_g}^{\dot{h}^*} & F_{\rho}^{\dot{h}^*} & F_{h}^{\dot{h}^*}
\end{bmatrix},$$

where the partial derivatives of the first three rows and columns are those in *(i)*-*(ix)* of matrix *J*, and the elements of the fourth row and column are

(x) $F_h^{\dot{l}_p^*} \equiv F_{14} = \left\{\left(\varphi_1^* - \gamma'^*\right)\left[\alpha\ \Psi_h^* - \Theta_h^*\left(1+l_p^*\right)^*\right]\right\} l_p^* > 0$

(xi) $F_h^{\dot{l}_g^*} \equiv F_{24} = \Theta_h l_g - \tau_1 \Psi_h^* - \gamma'^* \left[\alpha \Psi_h^* - \Theta_h^*\left(1+l_p^*\right)\right] l_g^* < 0$

(xii) $F_h^{\dot{\rho}^*} \equiv F_{34} = \omega_1^* \left[\alpha\ \Psi_h^* - \Theta_h^*\left(1+l_p^*\right)\right] > 0$

(xiii) $F_h^{\dot{h}^*} \equiv F_{44} = \left\{\nu'^*\ \Theta_h^* - \gamma'^* \left[\alpha \Psi_h^* - \Theta_h^*\left(1+l_p^*\right)\right]\right\} h^* < 0$

(xiv) $F_{l_p}^{\dot{h}^*} \equiv F_{41} = \left\{\nu'^*\ \Theta_{l_p}^* - \gamma'^* \left[\alpha \Psi_{l_p}^* - \Theta_{l_p}^*\left(1+l_p^*\right) - \Theta^*\right]\right\} h^* < 0$

(xv) $F_{l_g}^{\dot{h}^*} \equiv F_{42} = \left\{\nu'^*\ \Theta_{l_g}^* - \gamma'^* \left[\alpha \Psi_{l_g}^* - \Theta_{l_g}^*\left(1+l_p^*\right)\right]\right\} < 0$

(xvi) $F_{\rho}^{\dot{h}^*} \equiv F_{43} = \left\{\nu'^*\ \Theta_{\rho}^* - \gamma'^* \left[\alpha \Psi_{\rho}^* - \Theta_{\rho}^*\left(1+l_p^*\right)\right]\right\} h \gtrless 0$

In order to assess the stability properties of the system we check again the stability conditions stated by Routh theorem. In the case of a fourth-order differential equation system, the theorem requires that the coefficients of the characteristic equation $\lambda^4 + a_{1'}\lambda^3 + a_{2'}\lambda^2 + a_{3'}\lambda + a_{4'} = 0$ satisfy the following set of conditions:

[3.8] $a_{1'}, a_{2'}, a_{3'}, a_{4'} > 0$

[3.9] $a_{1'}a_{2'} - a_{3'} > 0$

[3.10] $a_{1'}a_{2'}a_{3'} - a_{1'}a_{1'} - a_{3'}a_{3'}a_{0'} > 0$

By Laplace expansion we get the following coefficients:

[3.11] $a_{1'} = -F_{11} - F_{22} - F_{33} - F_{44} = -Tr\left|J^H\right|$

[3.12]
$$\begin{aligned} a_{2'} = {} & \left[F_{11}F_{22} + F_{22}F_{33} - F_{31}F_{13} - F_{32}F_{23} - F_{21}F_{12} + F_{11}F_{33}\right] + \\ & \left[-F_{41}F_{14} - F_{42}F_{24} - F_{43}F_{34} + F_{44}F_{22} + F_{44}F_{33} + F_{44}F_{11}\right] \end{aligned}$$

[3.13] $a_{3'} =$

$$\left[-F_{11}F_{22}F_{33} - F_{12}F_{23}F_{31} - F_{13}F_{21}F_{32} + F_{31}F_{22}F_{13} + F_{33}F_{21}F_{12} + F_{32}F_{23}F_{11}\right] +$$
$$\left[\begin{array}{l} F_{41}\left(F_{14}F_{22} + F_{14}F_{33} - F_{24}F_{12} - F_{34}F_{13}\right) + \\ F_{42}\left(-F_{23}F_{34} - F_{14}F_{21} + F_{33}F_{24} + F_{24}F_{11}\right) + F_{43}\left(F_{22}F_{34} + F_{34}F_{11} - F_{31}F_{14} - F_{32}F_{24}\right) + \\ F_{44}\left(-F_{11}F_{22} - F_{22}F_{33} + F_{31}F_{13} + F_{32}F_{23} + F_{21}F_{12} - F_{11}F_{33}\right) \end{array}\right] =$$

[3.14] $a_{4'} =$

$$\begin{aligned}&\left[-F_{11}F_{22}F_{33}-F_{12}F_{23}F_{31}-F_{13}F_{21}F_{32}+F_{31}F_{22}F_{13}+F_{33}F_{21}F_{12}+F_{32}F_{23}F_{11}\right]+\\&F_{41}\left(-F_{12}F_{23}F_{34}-F_{13}F_{24}F_{32}-F_{14}F_{22}F_{33}+F_{32}F_{23}F_{14}+F_{33}F_{24}F_{12}+F_{34}F_{22}F_{13}\right)+\\&F_{42}\left(F_{11}F_{23}F_{34}-F_{13}F_{24}F_{31}+F_{14}F_{21}F_{33}-F_{31}F_{23}F_{14}-F_{33}F_{24}F_{11}-F_{34}F_{21}F_{13}\right)+\\&F_{43}\left(-F_{11}F_{22}F_{34}-F_{12}F_{24}F_{31}-F_{14}F_{21}F_{32}+F_{31}F_{22}F_{14}+F_{32}F_{24}F_{11}+F_{34}F_{21}F_{12}\right)+\\&F_{44}\left(F_{11}F_{22}F_{33}+F_{12}F_{23}F_{31}+F_{13}F_{21}F_{32}-F_{31}F_{22}F_{13}-F_{32}F_{23}F_{11}-F_{33}F_{21}F_{12}\right)\\&=\det\left|J^H\right|\end{aligned}$$

From these expressions we see that each coefficient $a_{i'},\ i=1,2,3$ can be expressed in terms of the analogous coefficients $a_i,\ i=1,2,3$ of the 3D characteristic equation previously examined, plus a term that is likely to have the appropriate sign for stability. By comparison with [2.16]-[2.18] we get:

[3.15] $a_{1'} = a_1 + [\textit{positive sign}]$

[3.16] $a_{2'} = a_2 + [\textit{positive sign}]$

[3.17] $a_{3'} = a_3 + [\textit{positive sign}]$.

The above indicate that the condition on the signs of the polynomial coefficients is more likely to be satisfied in the 4D system with the policy rule, than in the case of the 3D system, provided the term in square bracket in [3.15]-[3.17] is big enough. This is ensured when the partial derivatives related to the monetary policy variable $h$ $F_j^{\dot{h}*},\ j=l_p, l_g, \rho, h$ and $F_h^{\dot{s}*},\ s=l_p, l_g, \rho, h$ in *(x)-(xvi)*, are big enough.

The economic interpretation of the above is straightforward. Suppose in the 3D system [S.1] the stability condition [2.16] of a negative trace fails to be satisfied because of the economy's high propensity to incur in over indebtedness – i.e., $F_{l_p}^{\dot{l}_p*} > 0$ when $|\varphi_2|$ is very small. In this case the monetary rule can provide the required control: the condition $a_{1'} > 0$ in [3.8] can now be satisfied, provided $F_{44} = F_h^{\dot{h}*}{}^* < 0$ is strong enough and dominates.

Yet, the inclusion of a monetary policy rule in the 4D dynamical system does not ensure stability automatically. In fact, for the remaining two conditions [3.9]-[3.10] to be fulfilled, it is required that the stable feed-back chains activated by monetary policy, whose effect is captured by the partial derivatives $F_j^{\dot{h}*},\ j=l_p, l_g, \rho, h$ and

$F_h^{\dot{s}^*}$, $s = l_p, l_g, \rho, h$, dominate the unstable chains operating through a positive joint effect $F_{l_p}^{\dot{\rho}^*} F_{\rho}^{\dot{l}_p^*}$ and $F_{l_g}^{\dot{\rho}^*} F_{\rho}^{\dot{l}_g^*}$.[7]

When the latter effect is important, instability can be controlled by selecting an appropriate policy action capable of operating directly onto the source of instability. This can be provided by a countercyclical fiscal stimulus that made $F_{\rho}^{\dot{l}_g^*} > 0$, when $F_{l_g}^{\dot{\rho}^*} < 0$. In terms of the behavioural functions of our model, by choosing a value of $\beta_1$, the parameter for the fiscal policy stance, large enough. Hence, the control of instability requires implementing a policy aimed at sustaining aggregate expenditure when the economy's state of confidence is worsening.

Overall, just as the long-run dynamical behaviour of the model hinges on the resulting effect of stable and unstable feedback chains of the model, the effectiveness of stabilization policies depends on their impact on the real-financial mechanism at work in the model. So, monetary stabilization policy can result ineffective if the unstable expectations-expenditure channel dominates the stable interest rate-investment chain. In terms of the analysis above, satisfaction of condition [3.8] is not sufficient to ensure long-run stability. As the case with no policies, at the core of the long-run behaviour of system stands the out-of-equilibrium adjustment process that governs the state of confidence-indebtedness dynamics. The feed-back mechanisms at work in that dynamics crucially depend on the coefficients of the behavioural functions. The magnitude of these coefficients ultimately determine the dominance of stable or unstable dynamical chains, and the response of the system to the stabilisation policies put forward.

The following section contains the empirical analysis of the model, where we study the implications of our theoretical analysis with reference to the currency board experience of Argentina.

## 4. **Continuous-time empirical analysis**

### 4.1. **The short and long run**

The system [S.2] has the empirical counterpart for the policy analysis of stability in the following disequilibrium equations:

[4.1] $$\dot{y} = \varepsilon_1 \left( \gamma[\pi^e - i] - s + nx - il \right)$$

[4.2] $$\dot{i} = \delta \left( m^d - m^s \right)$$

---

[7] This can be checked by expliciting conditions [3.9] and [3.10] in terms of the characteristic equation coefficients given in [3.11]-[3.14].

[4.3] $$\dot{l}_p = \left[\left(\phi_1\left(\pi^e - i\right) - AVED\right) + \phi_2\left(l_p - AVLP\right) - \gamma[\pi^e - i]\right] l_p$$

[4.4] $$\dot{l}_g = \beta_0 + \beta_1\rho + \sigma y + il_g - \gamma\left[\pi^e - i\right] l_g$$

[4.5] $$\dot{\rho} = \left[\omega_1(\pi - i) - AVD\right] + \omega_2\left(l_p - AVL\right) + \omega_3 l_g$$

[4.6] $$\dot{m}^s = \dot{l} - i\,l + nx + \nu_m\left(i - \bar{i}\right) - \gamma[\pi^e - i]\left(m^s - l\right)$$

where now *AVED*, *AVLP*, *AVD* represent, respectively, the average values of expected net profitability, the stock of private debt and current profitability, $y_w$ world output, $x$ the multilateral real exchange rate, and $l = l_p + l_g$. The functional forms of equations [2.1], [2.6] and [3.2] are explicited in linear form as $\gamma[\pi^e\text{-}i] = \gamma_0 + \gamma_1\left(\pi^e\text{-}i\right)$, $m^d = \mu_1 y + \mu_2 i$, and $v(i - \bar{i}) = \nu_m(i - \bar{i})$, respectively. The monetary and fiscal policy parameters are here denoted by $\nu_m$ and $\beta_1$ , respectively. These, of course, are kept equal to zero at the beginning of our experiment, in order evaluate their specific contribution once activated. The nonlinearity of [4.1]-[4.6] is multiplicative and the coefficients considered represents the effects of the variables depicted in the theoretical equations. We stress our choice of recurring to a continuous time empirical analysis for it delivers as a natural outcome the eigenvalues and eigenvectors, and as such fits specifically with dynamical models where the main issue of interest is stability analysis. Moreover, such an approach is particularly suitable with macro data where there is no solution of continuity of the data-generating process over time. This allows to handle appropriately our model mixed stock-flow disequilibrium equations.

In order to study empirically the dynamics of the system [4.1]-[4.6], we have to choose between the alternative possibilities of linearizing around steady-state or the average value (Gandolfo 1981). Generally, average values are employed either when steady state values are not evaluable or because are not realistic from a policy perspective (Gandolfo 1993 and Wymer 1997). On the basis of Maggi et al. (2012) where we derived two equilibria characterized by very low and high interest rates, and so not usable for our policy analysis, we choose to perform the empirical analysis of the policies by having reference to average instead of steady-state values.

### 4.2 **Sensitivity of the system**

We now intend to develop an analysis on the effects of active stabilization policies. In particular, in line with the theory expounded, we are interested in analyzing the behavior of the model by relaxing the constraints imposed by the currency board arrangement, in terms of allowing for an autonomous monetary policy which means letting $\nu_m$ >0 in [4.6] for a wide range of policy coefficients. In a similar way, as regards fiscal policy, our experiment is to let for $\beta_1$ <0 in [4.4] for a wide choice of coefficients. This amounts to assuming a departure from the IMF policy recommendations for Argentina in the currency board years, recommendations oriented to stringent fiscal contractionary policies notwithstanding the severe recession under course[8]. Differently we perform a new counterfactual analysis by assuming that the successful prescription to restart the economy would have been to operate on the investment-expectation mechanism via appropriate anticyclical stimuli.

The empirical study we carry on consists in performing the sensitivity analysis using the algorithm for eigenvalues and eigenvectors developed in the C. Wymer procedure (Contines program, Wymer, 2005). Starting with eigenvalues we have

$$[4.7] \quad \frac{\partial \lambda_i}{\partial \mathbf{A}} = \left[ \frac{\partial \lambda_i}{\partial a_{jk}} \right] = h_i^* h_i^T$$

In eq. [4.7] matrix **A** is the coefficients matrix of the homogeneous linearized system [4.1]-[4.6] in normal form,[9] $\lambda_i$ are the eigenvalues, $h_i$* the $i$-th transposed row vector of the inverse eigenvector matrix and $h_i^{\,T}$ the $i$-th transposed column vector of the eigenvector matrix (a detailed proof is in Gandolfo (1981).

In order to highlight the role of the policy parameters we need to extend the previous formula to the structural form of our model, as follows

$$[4.8] \quad \frac{\partial \lambda_i}{\partial \vartheta_l} = \sum_j \sum_k \frac{\partial \lambda_i}{\partial a_{ik}} \frac{\partial a_{ik}}{\partial \vartheta_l}$$

Here $\vartheta_l$ are the structural parameters of the model among which are included $k_f$ and $k_m$, our policy parameters. These parameters are not estimable, because of the "experimental" nature of our analysis, in the sense that no actual policy action occurred in the direction we here assume. Therefore, the only possibility for an empirical analysis is to constraint the structural non-policy parameters to the coefficients values obtained in an estimation without the above-mentioned policies. To this aim we recur to the coefficients previously obtained in Maggi et al. (2012) by means of the non linear *exact discrete analogue* continuous-time-estimator (Escona program, Wymer (2005).

In Table 1 we report the implementation of formula [4.8]. The first column contains all the structural parameters, both estimated (all significant at 99% with correct sign) or calibrated. Of course, if exogenous the output of the sensitivity is just null. As aforementioned, for the calibrated parameters we tried a wide range of coefficients sorted first by sign for the different implications on the system's stability, and then by dropping those ones beyond which the effects on the eigensystem are similar, that is – in absolute value- around 30% both for $\beta_1$ and $\nu_m$. Such a choice seems particolarly reasonable also in consideration that the two coefficients are applied respectively to the change in the profit expectations (eq. [4.4]) and to the differential of the interest rate with the target (average) value, which are both small numbers.

[8] On this issue see IMF (2003), Perry and Servén (2003), Hausmann and Velasco A. (2002) and Daseking et al. (2004).

[9] As well known (Gandolfo, 2010) the stability of the endogenous variables of a vector **x** depends only on the coefficients of matrix **A** of the homogeneous system written in normal form, $\dot{\mathbf{x}}=\mathbf{Ax}$, where the exogenous variables of the corresponding structural system, in our case [4.1]-[4.6], are excluded.

Table 1. Sensitivity matrix of the eigenvalues with respect to the structural parameters.

| Parameter | Coefficient | $\lambda_1$=1.0102 (real) | $\lambda_2$= -0.0381 (real) | $\lambda_3$=-1.5033 (real) | $\lambda_4$=-2.7246 (real) | $\lambda_5$=-0.4316 (real) | imaginary part = ±0.0322 ($\lambda_5$) |
|---|---|---|---|---|---|---|---|
| $\gamma_1$ | 0.075 | -0.0402 | 0.2936 | -2.7395 | 2.2402 | 0.2255 | 3.8401 |
| $\mu_1$ | 1.44800 | -0.0061 | 0.0007 | -0.0021 | 0.027 | -0.0098 | 0.2327 |
| $\mu_2$ | -3.42400 | -0.0039 | 0.0032 | -0.0287 | 0.6361 | 0.0566 | 0.499 |
| $\varepsilon_1$ | 0.517 | 0.004255 | -0.00928 | 0.000967 | 0.011412 | -0.50368 | 2.863443 |
| $\delta$ | 0.72 | 0.0062 | -0.014 | 0.1322 | -2.9705 | -0.289 | -1.9051 |
| $\phi_1$ | 0.345 | -0.0003 | -0.0077 | -0.6329 | -0.1362 | -0.073 | -0.2936 |
| $\phi_2$ | -0.394 | 0.0001 | 0.0027 | 0.2049 | -0.0086 | 0.004 | 0.0154 |
| $\omega_1$ | 0.163 | -0.0024 | -0.2382 | 0.1388 | -0.1228 | 0.1123 | 0.3188 |
| $\omega_2$ | -0.427 | -0.0015 | -0.0426 | 0.0557 | -0.0032 | -0.0042 | -0.0115 |
| $\omega_3$ | -0.24 | -0.2254 | 0.0643 | 0.0541 | -0.0303 | 0.0686 | 0.2179 |
| $\sigma$ | -0.197 | -0.0217 | 0.0116 | -0.0068 | 0.0221 | -0.0026 | -0.6859 |
| $n_1$ | -0.253 | 0.002 | 0.0037 | -0.0062 | 0.0157 | 0.2509 | -2.0639 |
| $v_m$ | 0.3 | 0.0024 | -0.0042 | -0.0735 | 0.5155 | -0.1951 | -1.1440 |
| $\beta_1$ | -0.3 | -0.1919 | 0.2143 | 0.0066 | 0.0192 | -0.0241 | -0.0518 |

Now we comment on the results obtained in Table 1 underlying the empirical correspondence with the theory developed in sections 2 and 3. In particular, we focus the attention on the *leit-motive* of our model, that is, the endogeneity of the overborrowing-state of confidence interaction. In particular, the critical coefficients of our model, given the expectation-debt accumulation dynamics, are $\phi_2$, $\omega_1$, $\omega_2$, $\omega_3$, $\beta_1$, $v_m$ of equations [4.3]-[4.5]. As for $\phi_2$ we confirm a detrimental effect on stability deriving from a reduction in lenders response to firms degree of leverage. This is confirmed by the positive entries in Table 1 and in particular by the dominant positive change in $\lambda_3$. As for $\omega_1$, the coefficient linking the evolution of the state of confidence to current profits, the sensitivity shows that a dominant negative change in $\lambda_3$ occurs, which indicates that the stabilising effect dominates, and that the destabilising "inertia" of the model is now under control. As to $\omega_2$, the unstable dominant effect from a positive change on $\lambda_3$ is in line with the theoretical prescription that a reduction in expectations responsiveness to financial robustness increases instability. As to $\omega_3$, the sensitivity exhibits a dominant negative change on $\lambda_1$, the unique unstable eigenvalue. From the theoretical analysis above, we know that stability requires the joint effect $F_{l_g}^{\dot{\rho}^*} F_{\rho}^{i_g^*}$ to be negative. This result may be achieved when $\omega_3$ is increased in eq. (*viii*), so that $F_{l_g}^{\dot{\rho}^*} > 0$, together with $\beta_1 < 0$, so that in eq. (*vi*) $F_{\rho}^{i_g^*} < 0$, just coherently with the sensitivity analysis reported above. As to the increase in the coefficient related to monetary policy, $v_m$, it fails in controlling the instability of the system as indicated by the dominant detrimental effect on the most stable eigenvalue $\lambda_4$. According to the arguments developed above this may be explained in terms of the monetary policy inability to activate expenditure mechanisms via interest rates. As to the fiscal policy parameter $\beta_1$, a dominant detrimental effect on $\lambda_2$ confirms that a reduction in the fiscal policy stance increases instability. To further confirm the result on the role of an active countercyclical fiscal policy to control systemic instability in Argentina, we show in the following Table 2 that in the absence of active stabilization policies ($\beta_1$=$v_m$=0) the model exhibits a greater instability as indicated by the presence of two positive eigenvalues and greater oscillations.

Table 2. Null fiscal and monetary policies: $\beta_1$=$v_m$=0

| | Real part | Imaginary part | Modulus | Damping period | Period of cycle |
|---|---|---|---|---|---|
| $\lambda_1$ | 0.95313 | | 0.953 | | |
| $\lambda_2$ | 0.0287 | | 0.029 | | |
| $\lambda_3$ | -1.4685 | | 1.468 | 0.681 | |
| $\lambda_4$ | -2.8634 | | 2.863 | 0.349 | |
| $\lambda_5$ | -0.3846 | 0.1587 | 0.416 | 2.600 | 39.599 |
| $\lambda_5$ | -0.3846 | -0.1587 | 0.416 | 2.600 | 39.599 |

Other stabilizing dominant effects on eigenvalues $\lambda_5$ and $\lambda_4$, as easy to expect, are from positive changes in the speeds of adjustment $\varepsilon_1$ and $\delta$ of goods and money market equations respectively.

Turning now to the analysis of eigenvectors, we apply the algorithm [4.9] to eigenvector 1 associated to the unstable eigenvalue, in order to evaluate if it is possible to reduce to zero its detrimental effect. That is, if the system remains unstable despite the monetary and fiscal policies undertaken, we intend to investigate whether it may be leaded to a *controlled stability* by neutralizing the unstable eigenvalue with appropriate typologies of controls:

$$[4.9]\quad \frac{\partial h_i}{\partial \mathbf{A}} = \left[\frac{\partial h_i}{\partial a_{jk}}\right] = \sum_{j \neq i} a_{kl} C_{ij} \quad \text{with } C_{ij} = \frac{h_i^* h_j^T}{\lambda_i - \lambda_j}.$$

It is worthnoticing that the analysis of matrix [4.9] allows to account also for the effect of perturbing a null coefficients of matrix **A**. This amounts to considering the effect

from a switch-on of a new variable for the equation considered in the homogeneous system.
In the following Table 3 we focus our analysis on that element of eigenvector 1 noticeably different from zero and, as such, the primary source of instability. It refers to the general solution of the homogeneous system for the dynamics of $l_g$, the public debt. Moreover, among all the coefficients of matrix **A** the only relevant effects on the above element stem from coefficients $a_{j,\mathrm{lg}}$ ($j$=1,…6). This implies that a control on the model instability may be obtained by perturbing the coefficients of $l_g$ in each of the six homogeneous equations related to matrix **A**. This supports our strategy of finding a control on the effects of public debt. In the following Table 3 each column refers to the equation of the homogeneous system indicated by the first deponent of the perturbed coefficient.

Table 3. Sensitivity of the eigenvector 1

| eigenvector 1, element value: 0.972 | | | | | |
|---|---|---|---|---|---|
| $\frac{\partial h_1}{\partial a_{y,\mathrm{lg}}}$ | $\frac{\partial h_1}{\partial a_{i,\mathrm{lg}}}$ | $\frac{\partial h_1}{\partial a_{lp,\mathrm{lg}}}$ | $\frac{\partial h_1}{\partial a_{\rho,\mathrm{lg}}}$ | $\frac{\partial h_1}{\partial a_{\mathrm{lg},\mathrm{lg}}}$ | $\frac{\partial h_1}{\partial a_{m_s,\mathrm{lg}}}$ |
| -0.2546 | -0.1 | -0.2304 | 0.1558 | 0.0266 | 0.1252 |

As for the first equation, the control on output dynamics ($y$), operates through the effect on interest reimburse on public debt. Stability is improved by a perturbation of the related coefficient, which produces a reduction in the income leakage due to interest obligations. As for the second equation, the interest rate dynamics ($i$), $l_g$ is not present in the structure, and a switch-on of a positive coefficient lowers the interest rate, since in the context of our model, more debt is also more liquidity[10]. As for the equation for private debt dynamics ($l_p$), again $l_g$ it is not present in the structure, and a new control on $l_p$ through a reduction of $l_g$ (negative coefficient) means controlling for unstable cumulative indebtedness. As for fourth the equation ($\rho$), once again, controlling the effect of $l_g$ in the dynamics of $\rho$ amounts to controlling the unstable chain expectations-public debt. In the fifth equation ($l_g$), a reduction of the unstable feedback has the usual beneficial effect. As for the sixth equation ($m_s$), a reduction of the $l_g$ effects means a reduction in the drain of foreign reserves due to interest expenses.
Overall, the empirical analysis shows that a stabilizing policy centered on government spending represents the vehicle for controlling systemic instability. Due to its direct impact on the level of production, i.e., not mediated by interest rates, such a policy can control the unstable expectations-expenditure chains and thus systemic instability. As expected, the more consistent effects of a change in the coefficient of $l_g$ are those

[10] In effect the negative relationship between debt and interest rates is also found in different analytical frameworks, where it is proved that a liquidity premium is paid the more a debt is liquid. In this regards, see Goldreich et al.(2005), and Delle Chiaie and Maggi (2013).

relative to production (output, $a_{y,l_g}$) and the private financial sector ($a_{lp,l_g}$). This because the structure of the model is essentially demand-driven, with out-of-equilibrium adjustment in these markets, where the private sector bases its expectations on the capacity of firms to retrieve financial liquidity and where the task of the institutions should be that of preserving the liquidity itself by the choice of an "appropriate" currency arrangement.

## 5. Conclusions and further research

This research has been conducted with the aim of understanding the issues related to stabilization policies for an economy that chooses a hard peg arrangement, such as the ongoing euroization in EU countries, or the abandoned currency board for the case of Argentina. We develop a theoretical model on the base of a Minskyan set up, on which we perform a continuous time empirical analysis. Our results show that standard stabilization policies may turn to be counter-productive. In particular, macroeconomic instability stemming from overindebtness is bound to worsen with the implementation of a tight fiscal policy, since the latter adds to the fall in aggregate private spending driven by the vicious indebtedness-state of confidence circle. In such circumstances, a fall in interest rates promoted by accommodating monetary policies may be insufficient for economic recovery. The continuous time empirical approach we developed reveals particularly suitable for the issues examined, especially for equilibrium and stability analyses and for the study of the source and the cure of instability. However, further research with this approach is still to be done and encouraged, especially from the perspective of control problems applied to macroeconomic financial fragility.

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