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Dealing with unobservable common trends in small samples: a panel cointegration approach

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Abstract

Non stationary panel models allowing for unobservable common trends have recently become very popular. However, standard methods, which are based on factor extraction or models augmented with cross-section averages, require large sample sizes, not always available in practice. In these cases we propose the simple and robust alternative of augmenting the panel regression with common time dummies. The underlying assumption of additive effects can be tested by means of a panel cointegration test, with no need of estimating a general interactive effects model. An application to modelling labour productivity growth in the four major European economies (France, Germany, Italy and UK) illustrates the method.

Keywords: Panel cointegration, unobservable common factor, bootstrap, TFP.

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1 Introduction[1]

Non stationary panel models allowing for unobservable common trends have recently become very popular, for instance in the stream of literature devoted to the analysis of cross country growth differentials, where it coincides with the residual traditionally identified as total factor productivity. In empirical applications the unobserved common factor is estimated essentialy using either principal components (*e.g.*, Bai, Kao and Ng, 2009, henceforth BKN) or cross-section averages (Kapetanions, Pesaran and Yamagata, 2011, henceforth KPY). However, both approaches require relative large sample sizes in both panel dimensions. A simple alternative, somehow popular in the past (see, *e.g.*, Baltagi and Griffin, 1988) and recommended in the early panel cointegration literature as a way to reduce cross-section dependence (see, *e.g.*, Pedroni, 1999) is augmenting the panel regression with common time dummies. Now, this augmented panel model will be correctly specified only under the restrictive assumption that the unobservable trends of the various units differ only for a shift factor from the common trend (additive effects following the terminology of Bai, 2009). In the more general case in which the common trend is transmitted to that of each country by heterogenous loadings (interactive effects in Bai, 2009) the residuals of this augmented panel regression will not be stationary. This suggests that the validity of the additive effects assumption can be checked testing the implied hypothesis that the residuals of the augmented panel regression are stationary in all units of the panel. This task can be easily carried out using the bootstrap procedure for the maximum of the individual no cointegration statistics developed in Di Iorio and Fachin (2014). We now in section 2 outline the set-up using as a motivating example labour productivity growth in the four major European economies (France, Germany, Italy and UK), then in section 3 detail our proposal. In section 4 the proposed method is applied to the dataset discussed previously, and section 5 concludes.

2 Set-up

As an illustration consider Value Added per unit of labour inputs, briefly labour productivity, an economic indicator as simple as fundamental. The plot in Fig. 1 shows the evolution of this variable in the manufacturing industries of the four largest European economies, France, Germany, Italy and United Kingdom, over the period 1970-2007[2]. As we can see, in all cases

[1] Correspondance to: stefano.fachin@uniroma1.it. Research supported by MIUR PRIN grant 2010J3LZEN "Forecasting economic and financial time series: understanding the complexity and modelling structural change". All computations have been carried out using the free software package Gretl (http://gretl.sourceforge.net/).

[2] In 2007 these four countries accounted for 59% of the GDP of the European Union (source: elaborations on data from the stats.oecd.org database). The source of our data,

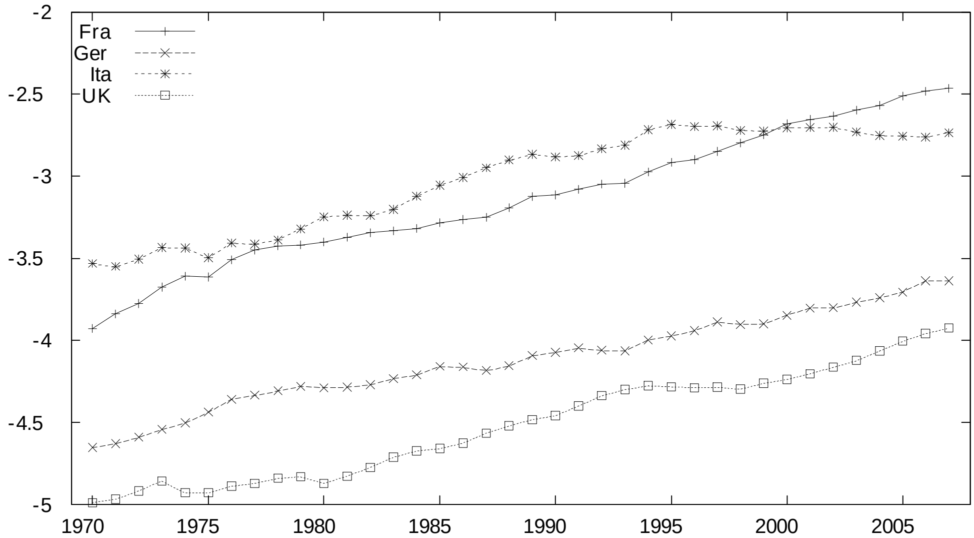

Figure 1: Log of Volume Indeces (1995=100) of manufacturing Value Added divided by total hours worked. Source: EU KLEMS Database.

productivity grew significantly over this time span, with average growth rates ranging between about 2%, in Italy, and 4%, in France[3]. Although growth appears to have been more regular in France, Germany and UK, while in Italy it somehow slowed down around 1990, the picture is broadly similar in the four countries.

The obvious question is how to explain these trends. The standard story is of course well-known, but as we will see it includes some delicate points. Assume for the sake of exposition Cobb-Douglas technology with constant returns to scale; generalisations to non-constant returns to scale and more general production functions are trivial. Using i and t as country and time indexes, denote by L_{it}, K_{it} and f_{it} respectively labour, capital, and an unobservable factor capturing the share of output growth not explained by observed inputs, i.e. total factor productivity (TFP) in the classical definition of residual ("A measure of our ignorance", Abramovitz, 1956). We can then write

$$\ln(Y_{it}) = \psi_i + (1-\beta_i)\ln(L_{it}) + \beta_i \ln(K_{it}) + u_{it} \tag{1a}$$

$$u_{it} = \mu_i + f_{it} + \omega_{it} \tag{1b}$$

where ω_{it} is an IID random noise. A similar set-up is the starting point of the analysis by Pedroni (2007), with the only difference that there the

EU-KLEMS, provides data only up to 2007, thus not covering the 2008 recession. Since modelling such a troublesome period at the very end of the sample is not advisable this limitation is not as severe as it may appear at a first sight.

[3] More details in Table 1, Section 4.

unobservable TFP trend is replaced by a linear time trend. Setting $\pi_{it} = \ln(Y_{it}/L_{it})$, $k_{it} = \ln(K_{it}/L_{it})$ and rearranging the first equation yields

$$\pi_{it} = \psi_i + \beta_i k_{it} + u_{it} \tag{2a}$$

Substituting for the error u_{it} we eventually have

$$\pi_{it} = \alpha_i + \beta_i k_{it} + f_{it} + \omega_{it}. \tag{3}$$

where $\alpha_i = \psi_i + \mu_i$. Labour productivity growth in country i is thus explained by growth of the capital/labour ratio and by a TFP index measuring Hicks-neutral technical progress. Since for many countries growth accounting estimates of TFP are available, either from national statistical agencies or from international sources such as the project EU KLEMS (Timmer *et al.,* 2010), a first option would be plugging one of such estimates of f_{it} into (3) and estimate it separately for each individual country. However, the cross-country perspective, often of considerable interest, will in this way be lost. In fact, models like (3) are widely used in the literature on cross-country growth empirics started by Mankiw, Romer, and Weil (1992), and typically estimated on panels of countries imposing some restriction, *e.g.*: coefficients homogeneity (leading to pooled estimation), stationarity (leading to use of fixed effects or GMM estimators), cross-section independence (which justifies a variety of popular estimation strategies). As pointed out by Eberhardt and Teal (2011), most of these restrictions are often at odds with the properties of the datasets examined, and owe their popularity only to their convenience. Our aim here is precisely the opposite, *i.e.* designing an estimation strategy starting from the features of our data. In our case, even the simple visual inspection of the data suggests non-stationarity and cross-country dependence as two properties that cannot be ignored. The former is easily acknowledged by application of cointegration modelling procedures. The latter suggests the assumption that technical progress in each country is driven by an unobservable common trend (say f) to be a *prima facie* plausible restriction for this panel of closely integrated, advanced economies. We thus consider the following representation of the latent TFP trends:

$$f_{it} = \lambda_i f_t + \zeta_{it}$$

implying for the model error

$$u_{it} = \mu_i + \lambda_i f_t + (\zeta_{it} + \omega_{it}) \tag{4a}$$

so that the productivity equation is

$$\pi_{it} = \alpha_i + \beta_i k_{it} + \lambda_i f_t + v_{it} \tag{5}$$

where $v_{it} = \omega_{it} + \zeta_{it}$.

Our key question has now became how to estimate equations (2a)-(4a), or equivalently, equation (5). If the the primary aim is obtaining good estimates of the elasticities β_i a simple solution is the CCE estimator by KPY, which entails augmenting the productivity equation with cross-section averages of dependent and independent variables. If instead the common trend f is of interest in itself the more common solution is well represented by BKN, who suggest an iterative procedure in which f_t would be estimated (conditionally on the $\widehat{\beta}' s$) as the first eigenvector of the matrix $(1/NT^2)\sum_{i=1}^{N}\mathbf{u}_i\mathbf{u}_i'$, where $\mathbf{u}_i$ is the $T\times 1$ vector of residuals of equation (2a) for unit i. Although there are some differences among the two approaches (KPY's appears to be overall more flexible, as BKN assume known number of factors, homogenous coefficients and no long-run links between f and X), in both cases the problem caused by the presence of the unobserved factor f_t is solved relying on asymptotic arguments in one or both dimensions. More precisely, the mean square estimation error of the latent factors is shown by KPY to converge to zero as $N \to \infty$ for given T when the number of factors is smaller than that of the explanatory variables, and by BKN to be of order $O(1/N) + O(1/T)$.

Further, the ratio between the two sample sizes may matter. Urbain and Westerlund (2011) show that with stationary data estimating (1a) augmented with an estimate of the common factor of either type (PC or cross-section average) delivers estimators with bias $O(\sqrt{T/N})$, vanishing with the time sample size T only if $N > T$. Hence, we should expect relatively large N to be needed for good results; in fact, neither BKN nor KPY report simulation results for $N < 20$. We thus have to conclude that for a panel like ours, with a cross-section sample size $N = 4$, much smaller than the time sample size, $T = 38$, some other approach should be seeked.

3 Modelling latent trends using time dummies

Following the early contribution by Baltagi and Griffin (1988) and building on Fachin and Gavosto (2010), consider the simple solution of the estimation problem described at the end of the previous section given by augmenting the panel regression with common time dummies, a common practice in the "small T, large N" literature[4]. Following standard notation let $\mathbf{k}_i = \begin{bmatrix} k_{i1} & \dots & k_{iT} \end{bmatrix}'$, $\mathbf{0}_{T\times 1}$ a column vector of zeroes of length T, $\mathbf{I}_{T\times T}$

[4]Common time dummies are also used by Islam's (1995) model with 5-years averages, but the common TFP trend is implicitly treated as a nuisance factor and as such not discussed.

an identity matrix of dimension T, and

$$\begin{aligned}
\boldsymbol{\pi}_{NT\times 1} &= \begin{bmatrix} \pi_{11} & \dots & \pi_{1T} & \dots & \pi_{N1} & \dots & \pi_{NT} \end{bmatrix}', \\
\boldsymbol{\alpha}_{NT\times 1} &= \begin{bmatrix} \alpha_1 & \dots & \alpha_1 & \dots & \alpha_N & \dots & \alpha_N \end{bmatrix}', \\
\mathbf{k}_{NT\times N} &= \begin{bmatrix} \mathbf{k}_1 & \mathbf{0}_{T\times 1} & \dots & \mathbf{0}_{T\times 1} \\ \mathbf{0}_{T\times 1} & \mathbf{k}_2 & & \mathbf{0}_{T\times 1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_{T\times 1} & \mathbf{0}_{T\times 1} & \dots & \mathbf{k}_T \end{bmatrix}, \mathbf{D}_{NT\times T} = \begin{bmatrix} \mathbf{I}_{T\times T} \\ \vdots \\ \mathbf{I}_{T\times T} \end{bmatrix}.
\end{aligned}$$

Then an heterogenous panel model based on (2a) augmented with common time dummies will be

$$\boldsymbol{\pi} = \boldsymbol{\alpha} + \mathbf{k}\boldsymbol{\beta} + \mathbf{D}\boldsymbol{\phi} + \mathbf{v}. \tag{6}$$

where $\boldsymbol{\phi} = \begin{bmatrix} \phi_1 & \dots & \phi_T \end{bmatrix}'$ is the $T \times 1$ vector of coefficients of the common time dummies. For unit i at time t we thus have

$$\pi_{it} = \alpha_i + \beta_i k_{it} + \phi_t + v_{it} \tag{7}$$

which is equivalent to (5) under the assumption

$$f_{it} = \eta_i + f_t + v_{it}.$$

That is, the heterogenous log TFP trends f_{it} deviate from a common trend f_t only by a (log) shift factor (not identified, as empirically it cannot be separated from the fixed effect α_i). Under this assumption, which Bai (2009) calls "additive effects", the growth rates of the unit-specific unobservable trends and those of the common trend are identical, while under the interactive effects hypothesis with non-unit loadings they will differ.

Obviously, the key empirical question is how to test the restriction of additive effects. Bai (2009) suggests an Hausman-type test, which however has the considerable drawback of requiring estimation of the model under both hypothesis. We propose a completely different approach, which requires only estimation of the restricted additive effects model. Recalling that we are assuming all variables to be non stationary, if this restriction is not valid for all units the panel model (6) will not be a long-run equilibrium relationship, and its residuals will be non stationary. Hence, to accept the validity of the additive effects assumption we need to reject H_0 : "no cointegration in all countries" in favour of H_1 : "cointegration in *all* countries". Denote by θ_i the no cointegration statistic computed for unit i, and with no loss of generality assume it to be the popular Engle-Granger test. Since in order to reject H_0 in favour of H_1 we require cointegration to hold in all units of the panel, the panel statistic we need to use is the individual statistic most favourable to H_0. Given that the rejection region of the Engle-Granger test is the left tail, this is obviously the highest of the individual statistics,

$Max(\theta_i)$. To the best of our knowledge only two testing procedures based on this statistic have been proposed so far: the IV test by Chang and Nguyen (2012) and a bootstrap procedure by DIF. The IV test can in principle be applied to dependent panels, but under long-run dependence it relies on sets of instruments not invariant to the ordering of the units[5]. The authors remark that although this may have some impact on the small sample results, so that the results are not guaranteed to be invariant to the ordering of the units. This is definitely a undesirable feature. Further, simulation results reported by DIF suggest that in empirically relevant conditions (small time samples, common factors in the variables) the IV panel cointegration test based on a mean of the individual no cointegration statistics is affected by considerable size distortion. Result not reported here for reasons of space, but available on request, confirm this holds for the maximum as well. On the contrary, DIF's simulations show that the bootstrap test for the maximum of the individual statistics has good size and power properties. Its asymptotic properties are not discussed by DIF, but are readily obtained from the results they provide (for the reference case of independent units) for the analogous procedure for $Mean(\theta_i)$.

The argument is the following. First of all, Proposition 2 in the Appendix of DIF states that the no cointegration statistics computed on the boostrap pseudodata for unit i will have the same limiting distribution of the empirical no cointegration statistic. Invoking the continuos mapping theorem (CMT), DIF's Proposition 3 extends this result to the mean of statistics computed on independent units. Now, recall that the Engle-Granger statistic θ_i is a continuous function. Since the maximum among continuos functions is itself a continuos function, then the CMT can be invoked also in this case. It thus follows that DIF's Proposition 3 can be extended to $Max(\theta_i)$. In other terms, the bootstrap test for $Max(\theta_i)$ put forth in DIF is asymptotically valid for the reference case of independent units.

Summing up, our proposal is to carry out the estimation and testing task in an iterative fashion, with the following steps:

1. estimate by OLS an heterogenous panel labour productivity model with common time dummies, *e.g.* $\boldsymbol{\pi} = \boldsymbol{\alpha} + \mathbf{k}\boldsymbol{\beta} + \mathbf{D}\boldsymbol{\phi} + \mathbf{v}$;

2. test for non-stationarity the residuals $\widehat{v}_{it}, i = 1, \dots, N, t = 1, \dots, T$, applying the panel cointegration bootstrap test for H_0 : "no cointegration in all units" against H_1 : "cointegration in all units" by DIF, based on the maximum of the individual Engle-Granger type statistics;

3. if cointegration holds in all units, recover the common TFP trend as the estimates $\widehat{\boldsymbol{\phi}}$ of the coefficients of the time dummies D_t.

[5]More precisely, the instruments to be used in each unit are Hermite functions of the variables of the other units, with the order of the polynomial a function of the unit index.

4. estimate models for the deviations of labour productivity from the estimated common TFP trend $\widetilde{\pi}_{it} = \pi_{it} - \widehat{\phi}_t$.

Although the four steps are in principle rather obvious, some remarks are in order. If the specification used in step 1 (*i.e.*, CES or Cobb-Douglas, constant or unconstrained returns to scale, etc.) is correct the procedure is completed with step 4. However, with small time samples some specification search may be required, as the presence of redundant regressors may destroy cointegration[6]. Note that if cointegration does not hold we have no basis for formal inference, and this search will have to be done heuristically, excluding variables with coefficients with wrong signs or very large standard errors. Estimates of the variances of coefficient estimates are not available for the panel model estimated by OLS, and Di Iorio and Fachin (2012) showed that system estimators such as FM-SUR deliver performances typically inferior to single-equation ones. The best option thus seems to model separately for each country the deviations of labour productivity from the common TFP trend as estimated in the fixed effect panel equation. In this way it is possible to use an efficient single-equation estimator, such as FM-OLS, iterating steps 1-4 until the hypothesis "no cointegration in all units" is rejected in step 2. Overall, the proposed procedure appears a relatively simple and robust way to account for a common unobservable trend.

Before moving in the next section to its application to the labour productivity data discussed above, a remark is in order. The additive effects hypothesis entails the following multiplicative level model ($\Pi = Y/L, U = e^u, V = e^v, F = e^f$):

$$\Pi_{it} = e_{it}^{\alpha_i} \left(\frac{K_{it}}{L_{it}} \right)^{\beta_i} U_{it} \tag{8a}$$

$$U_{it} = F_t V_{it} \tag{8b}$$

so that

$$\Pi_{it} = \left[e^{\alpha_i} \left(\frac{K_{it}}{L_{it}} \right)^{\beta_i} V_{it} F_t \right] \tag{9}$$

while Bai's model yields

$$\Pi_{it} = \left[e^{\alpha_i} \left(\frac{K_{it}}{L_{it}} \right)^{\beta_i} V_{it} F_t^{\lambda_i} \right].$$

[6] This obvious, yet largely overlooked point, is easily seen. Consider three $I(1)$ variables, y_t, x_t and z_t, such that the Data Generating Process (DGP) is $y_t = \beta x_t + \varepsilon_t$, where ε_t is $I(0)$. Let the estimated model be $y_t = b x_t + c z_t + e_t$. Then substituting from the DGP into the model for y_t we obtain $\beta x_t + \varepsilon_t = b x_t + c z_t + e_t$, so that the residuals of the estimated model are $e_t = (b - \beta) x_t + c z_t + \varepsilon_t$. Now, asymptotically $b \to \beta, c \to 0$, so that e_t will converge to the true stationary errors ε_t. However, in small samples c might be in practice different from zero, so that the estimated residuals will contain the non-stationary component $c z_t$.

In either cases, F is a common factor influencing growth in all countries, naturally interpreted as the trend in technical progress (in a wide sense) common to the entire group of countries. On the other hand, growth accounting applied to country aggregates, such as the EU, produce estimates of the latent trend in the aggregate model (say, F_t^A)

$$\Pi_t = \left[e^{\alpha} \left(\frac{K_t}{L_t} \right)^{\beta} V_t \right] F_t^A \tag{10}$$

To compare the two latent trends, first of all consider that $\Pi_t = \sum_{i=1}^{N} \left(\frac{L_{it}}{L_t} \right) \Pi_{it}$ and $\frac{K_t}{L_t} = \sum_{i=1}^{N} \frac{L_{it}}{L_t} \frac{K_{it}}{L_{it}}$. Hence, (10) can be rewritten as

$$\sum_{i=1}^{N} \left(\frac{L_{it}}{L_t} \right) \Pi_{it} = \left[e^{\alpha} \left(\sum_{i=1}^{N} \frac{L_{it}}{L_t} \frac{K_{it}}{L_{it}} \right)^{\beta} V_t \right] F_t^A$$

In order to appreciate more easily the key difference between the two latent trends assume labour shares to be constant over time and units, so that $L_{it}/L_t = \eta$ for each i, t. We then have

$$\sum_{i=1}^{N} \Pi_{it} = e^{\alpha} \eta^{\beta - 1} \left(\sum_{i=1}^{N} \frac{K_{it}}{L_{it}} \right)^{\beta} V_t F_t^A$$

so that

$$F_t^A = \frac{\sum_{i=1}^{N} \Pi_{it}}{e^{\alpha} \eta^{\beta-1} \left(\sum_{i=1}^{N} \frac{K_{it}}{L_{it}} \right)^{\beta} V_t}. \tag{11}$$

On the other hand, summing (9) over units and rearranging yields

$$F_t = \frac{\sum_{i=1}^{N} \Pi_{it}}{\sum_{i=1}^{N} e^{\alpha_i} \left(\frac{K_{it}}{L_{it}} \right)^{\beta_i} V_{it}} \tag{12}$$

Comparing (11) and (12) we can appreciate that in the first case the key factor is the contribution coming from the aggregate of individual capital/labour ratios, $\left(\sum_{i=1}^{N} \frac{K_{it}}{L_{it}} \right)^{\beta}$, while in the second case the key factor is the aggregate of the individual contributions, $\sum_{i=1}^{N} \left(\frac{K_{it}}{L_{it}} \right)^{\beta_i}$. Summing up, there is no obvious relationship between the latent trend estimated on the aggregate data, F^A, and that estimated on the panel data, F.

4 Modelling labour productivity

As we have seen above, labour productivity followed a growing trend in all countries examined. However, a closer look at the data (details in Table

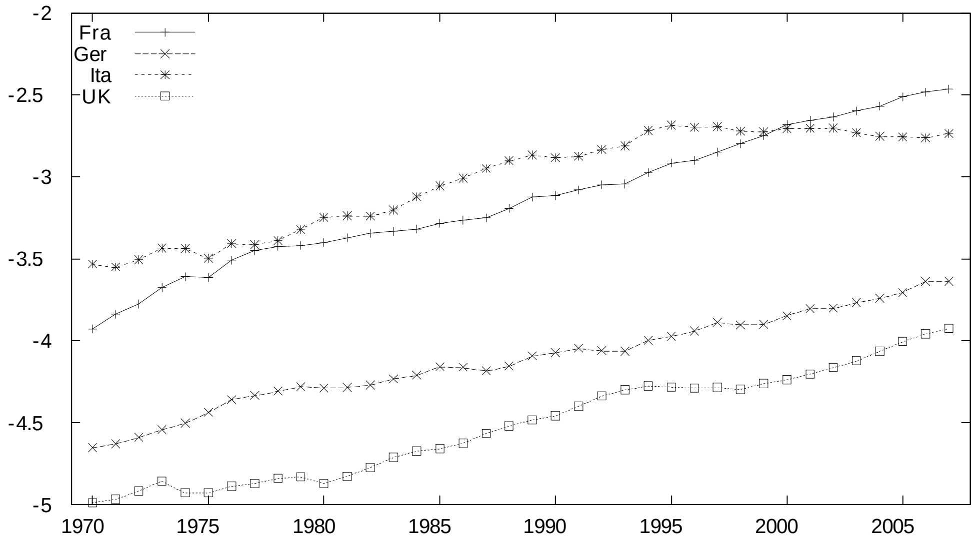

Figure 2: Log of Volume indeces (1995=100) of capital services in the manufacturing industry divided by total hours worked. Source: EUKLEMS Database.

1) reveals some heterogeneity. More precisely, while in the second part of the sample the growth rate slowed down slightly in France and Germany and sizeably in Italy, it even accelerated in the UK. Very much the same holds for the capital-labour ratio (Fig. 2), as measured by the ratio of the volume indices of capital services and total hours worked, with the exception that this variable slowed down in the UK as well. The cross-plot of average rates of growth of the capital/labour ratio and labour productivity (Fig. 3) confirms that the association between productivity growth and capital deepening has been rather close. The slow-down in capital deepening is thus a plausible culprit for the productivity slowdown in the second half of the sample.

Finally, it is interesting to see that although labour inputs (Fig. 4) declined substantially everywhere, there is some considerable heterogeneity. In France, Germany and UK the fall has been continuous since 1970, with final values at most half the 1970 ones[7]. Italy followed again a peculiar path, as hours worked kept increasing until 1979, then shrunk. As a result, the 2007 figure was, albeit marginally, still higher than that of 1970.

[7] These figures are in line with those for the EU (10 member states), where labour inputs fell about one third between 1980 and 2005: see Timmer *et al.* (2010), p. 33.

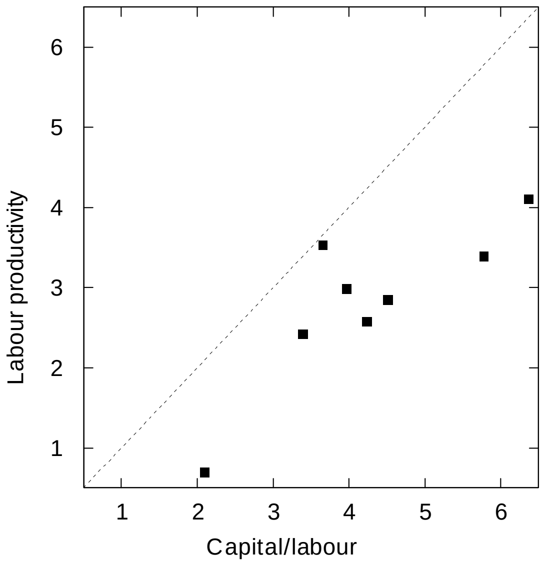

Figure 3: France, Germany, Italy and UK: labour productivity and capital/labour ratio in the manufacturing industry, average rates of growth 1970-1989 and 1990-2007, with 45° line. Source: elaborations on EU KLEMS data.

Table 1
Labour Productivity and Capital/Labour ratio
growth in the Manufacturing Industry
Average annual rates of growth×100

	France		Germany		Italy		UK	
	Y/H	K/H	Y/H	K/H	Y/H	K/H	Y/H	K/H
1970-1989	4.10	6.37	2.84	4.51	3.39	5.78	2.57	4.24
1990-2007	3.53	3.66	2.42	3.40	0.69	2.10	2.98	3.97
1970-2007	3.92	5.17	2.70	4.07	2.12	4.07	2.84	4.22

Y/H: (volume index of value added)/hours worked
K/H: (volume index of capital services)/hours worked
Source: elaborations on EU KLEMS data.

Let us now turn to model estimation. We select the specification as suggested in the previous section, starting with the Kmenta (1967) linearisation of the Constant Elasticity of Substitution (CES) production function:

$$\pi_{it} = \alpha_i + \gamma_i l_{it} + \beta_{1i} k_{it} + \beta_{2i} k_{it}^2 + v_{it} \tag{13}$$

The simpler Cobb-Douglas is obtained setting $\beta_{2i} = 0$, while setting $\gamma_i = 0$

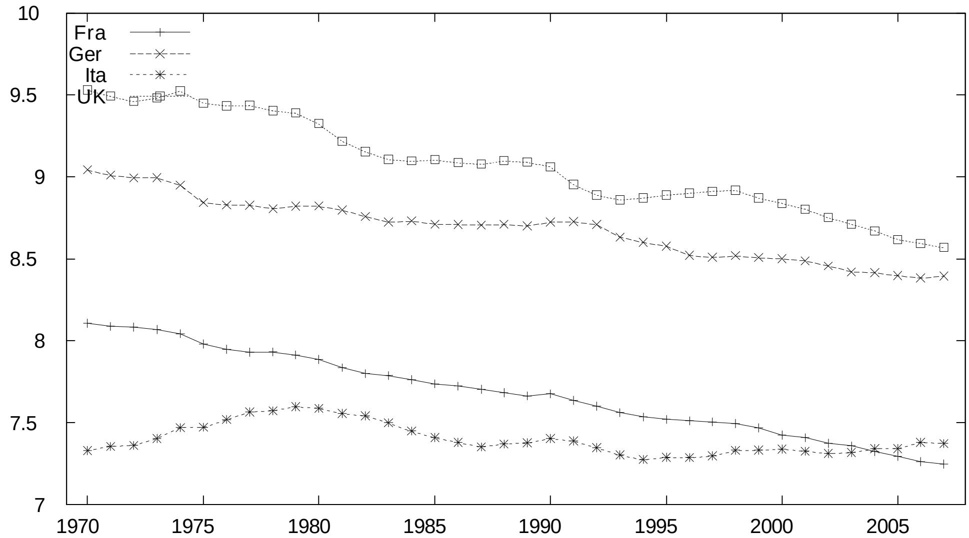

Figure 4: Log of hours worked in the Manufacturing industries, 1970-2007. Source: EU KLEMS Database.

imposes constant returns to scale. The evaluation of the time series properties of the variables, reported in Table 2, suggests that constant returns to scale may be appropriate in at least two cases. In the top part of Table 3 we report the combination of specifications that delivered the lowest p-value for the maximum of the individual statistics[8], which is still much larger than any conventional threshold (20.9%). Further, as it can immediately appreciated, many estimates are essentially meaningless (*e.g.*, the strongly increasing and decreasing returns to scale in UK and France). We thus conclude that the additive TFP hypothesis is not compatible with the data. A possible explanation is the presence of Italy, which was seen to have followed a somehow idiosyncratic path. We thus repeat the estimation with a panel including only France, Germany and UK. The results (lower part of Table 3) are now reasonably in favour of cointegration in all countries: the p-value of $Max(\theta_i)$ is 13.6%, and the FM-OLS estimates plausible, suggesting that returns to scale are constant in France and UK and decreasing in Germany. In the only case of a Cobb-Douglas, the coefficient of the capital/labour ratio is in line with the one-third value taken as a benchmark in the literature (see, *e.g.*, Mankiw, Romer and Weil, 1992). The estimated common TFP trend is plotted in Fig. 6 along with the EU-KLEMS growth accounting estimates for each country, available for the entire sample for UK and since 1980 and 1991 respectively for France and Germany. Taking into account the limited information for the last two countries, the comparison

[8]Note that the statistics actually computed are slightly different from the standard Engle-Granger test. For details see Di Iorio and Fachin (2014).

is rather striking: the dynamics of the three country TFP trends do seem to be largely explained by that of our estimate of the common TFP trend. This is confirmed by growth rates, plotted in Fig. 7. cConsistently with the addictive effects hypothesis. but for isolated exceptions the growth patterns of the country-specific trends and the common trend are essentially the same.

Table 2
ADF-GLS Unit root and cointegration tests

	France	*Germany*	*Italy*	*UK*
Y/L	-1.83	-2.03	-1.02	-2.26
K/L	-0.83	-1.14	-0.75	-3.10*
L	-3.69**	-2.79	-2.04	-3.39**

ADF-GLS tests with constant and trend,
lag length selected by AIC, max lag 2;
*,**: significant at 10%,5%.
Variables in logs.

Table 3
Modelling Labour Productivity

Panel A France, Germany, Italy, UK				
$Max\ EG\ (100 \times p - value)$ -2.24 (20.9)				
FM-OLS estimates				
	France	*Germany*	*Italy*	*UK*
γ	-2.62 (-4.99)	-0.65 (-1.51)	-0.50 (-4.96)	4.49 (1.74)
β_1	0.37 (-1.46)	0.53 (2.90)	0.80 (36.28)	3.17 (1.98)
β_2	–	–	–	–
Panel B France, Germany, UK				
$Max\ EG\ (100 \times p - value)$ -2.88 (13.6)				
FM-OLS estimates				
	France	Germany	UK	
γ	–	-0.63 (-1.81)	–	
β_1	2.65 (6.37)	0.37 (2.54)	2.29 (2.24)	
β_2	0.29 (4.59)	–	0.19 (1.66)	

Bootstrap: 5000 redrawings, average block size $1.75\sqrt{T}$.
FM-OLS: dependent variable: deviations of log labour productivity from estimated common TFP trend; symbols, see equation (13); t-statistics in brackets.

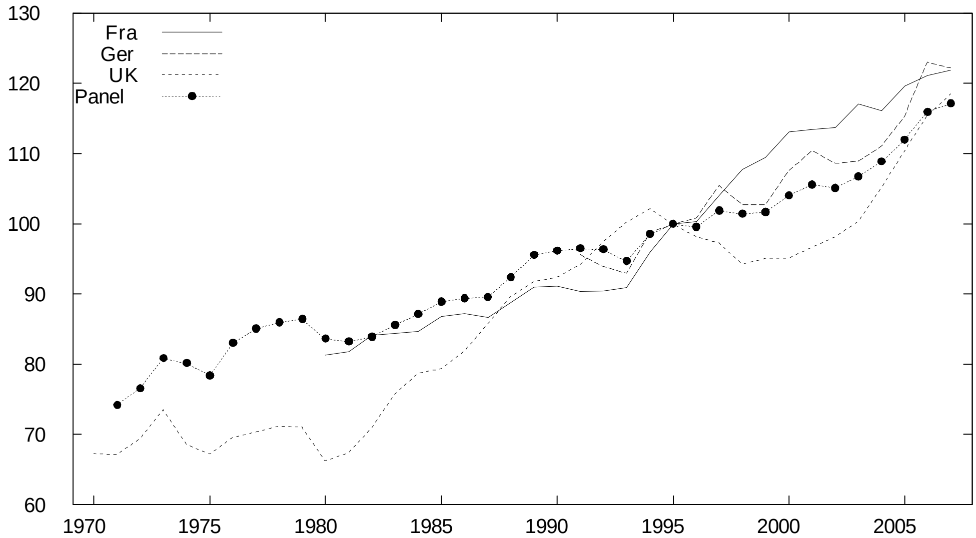

Figure 6: Panel and growth accounting TFP estimates (France from 1980, Germany from 1991), 1995=100. Source of growth accounting estimates: EU-KLEMS Database.

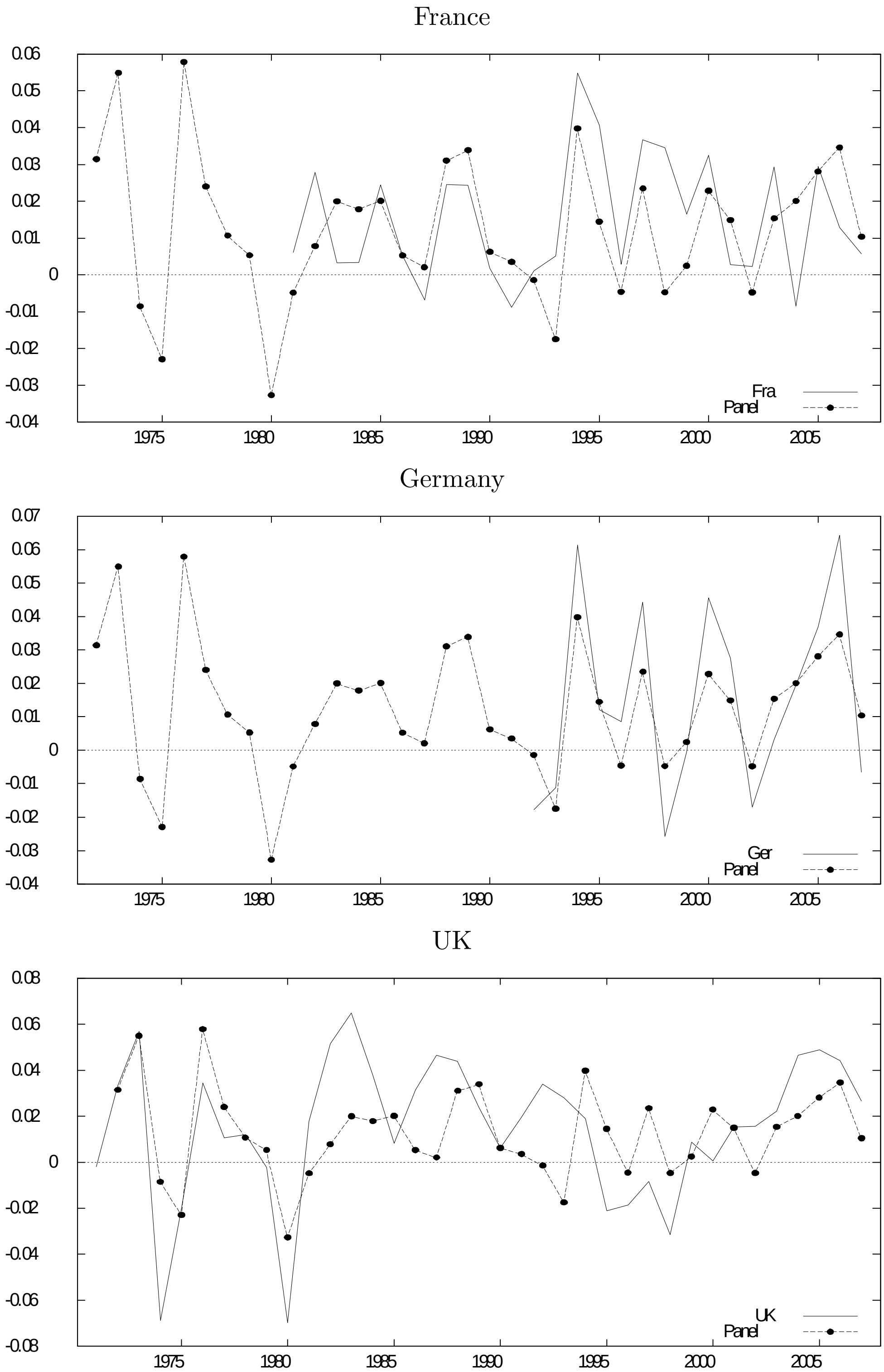

Figure 7: Panel and growth accounting TFP estimates, log differences. Source of growth accounting estimates: EU-KLEMS Database.

5 Conclusions

We started with a very simple question: how can panel models with unobservable commong trends be applied in practice to datasets with small time samples and possibly very small cross-section sample size? Standard methods to deal with common factors are of asymptotic nature, and Urbain and Westerlund (2011) indeed show that their performance can be disappointing with small samples. We propose an extremely simple solution, namely using a panel regression with common time dummies. This will be correctly specified only for the restrictive case of additive effects, *i.e.* when the idiosyncratic trends are obtained adding a shift factor to the common trend. We suggested to test this restriction using a panel cointegration test powerful against the alternative of cointegration in all units of the panel, such as one based on the maximum of Engle-Granger statistics. A bootstrap procedure of this type put forth in Di Iorio and Fachin (2014) and shown there by simulation to have good size and power properties is argued here to be asymptotically valid. Note that in this way there is no need to estimate the more general interactive effects model. Applying this approach to data on the manufacturing sectors of the four largest European economies (France, Germany, Italy and UK) we could reach the conclusions that TFP dynamics in France, Germany and UK has been essentially driven by the same common trend, while Italy followed an idiosyncratic path.

6 References

Abramovitz, M. (1956) "Resource and output trend in the United States since 1870" *American Economic Review*, 46, 5-23.

Bai, J. (2009) "Panel data models with interactive fixed effects" *Econometrica*, 77, 1229-1279.

Bai, J., C. kao, S. Ng (2009) "Panel cointegration with global stochastic trends" *Journal of Econometrics*, 149, 82-99.

Baltagi, B.H. and J.M. Griffin (1988) "A general index of technical change" *Journal of Political Economy*, 96, 20-41.

Chang Y. and C.M. Nguyen (2012) "Residual based tests for cointegration in dependent panels" *Journal of Econometrics*, 167, 501–520.

Di Iorio, F. and S. Fachin (2012) “A Note on the estimation of long-run relationships in panel equations with cross-section linkages” *Economics: The Open-Access, Open-Assessment E-Journal*, 6, 2012-20. http://dx.doi.org/10.5018/economics-ejournal.ja.2012-20

Di Iorio, F. and S. Fachin (2014) "A panel cointegration study of the long-run relationship between savings and investments in the OECD economies, 1970-2007" *Empirical Economics*, 46, 1271-1300.

Eberhardt, M. and F. Teal (2011) "Econometrics for grumblers: a new look at the literature on cross-country growth empirics" *Journal of Economics Surveys,* 25, 109-155.

Fachin, S. and A. Gavosto (2010) "Trends of labour productivity in Italy: a study with panel co-integration methods" *International Journal of Manpower,* 31, 755-769.

Kapetanios, G., H.M. Pesaran and T. Yamagata (2011) "Panels with non-stationary multifactor error structures" *Journal of Econometrics*, 160, 326–348.

Kmenta, J. (1967) "On the Estimation of the C.E.S. Production Function" *International Economic Review,*2, 180-189.

Mankiw, N. G., D. Romer, and D. Weil (1992) "A contribution to the empirics of economic growth," *Quarterly Journal of Economics*, 107, 407-37.

Pedroni, P. (1999) "Critical values for cointegration tests in hetergenous panels with multiple regressors" Oxford Bulletin of Economics and Statistics, 61, 653-670.

Pedroni, P. (2007) "Social capital, barriers to production and capital shares: implications for the importance of parameter heterogeneity from a nonstationary panel approach" *Journal of Applied Econometrics*, 22, 429–451.

Timmer, M.P., R. Inklaar, M. O'Mahony and B. Van Ark (2010) *Economic Growth in Europe* Cambridge University Press, Cambridge.

Urbain, J.P. and J. Westerlund (2011) "Cross sectional averages or principal components?" METEOR RM/11/053 Maastricht University.