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Abstract

With this study we intend to define a methodology capable to deal with the task of evaluating and planning the interdependent dynamics of growth for some European countries together with their foreign partners. To that aim we employ a nonlinear differential equations system representing a disequilibrium model based on a Schumpeterian evolutionary context with endogenous technology. We use such a model in order to disentangle the interrelationships occurring among countries for the critical variables considered. That is, we succeed in evaluating the contribution to growth of a country with respect to another one in terms of the variables involved. We address and corroborate the validity of our conjectures on the importance of the business services in the innovation and production processes by presenting also a minimal model. Further, we provide an evaluation of the critical variables for policy purposes. We then perform a sensitivity analysis to assess per each country the effectiveness of some possible efforts in order to gain stability.

JEL: C33, C62, C61, O11, O33, O34

Keywords: Continuous Time Panel Econometrics, Distance, Programming, Growth, Stability, Sensitivity, Technology, Business Services.

1. Introduction

This paper shows how to consider in a structural, and possibly general, equilibrium context a complete dynamical analysis of an endogenous growth model with diffusion. Notably, these two issues have been dealt with in the literature under the compromise that only one of the two might be addressed satisfactorily. That is, either some aspects of the dynamics are usually missed when the structural analysis is detailed or the opposite occurs. In particular, Eaton and Kortum (1999) emphasize the difficulty to deal with both these aspects and concentrate on a detailed description of the innovation diffusion process. Actually, they provide a complete description of such a process by making use of the patents applications data from the WIPO data base. Other literature focuses the attention on the dynamics while neglects the diffusion problems as in Jones (1995) and in the following strand of the New Economic Geography based on the seminal work of Krugman (1991a, b), Venables (1996) Englmann and Walz (1995), Walz (1996) in a two-region framework. In these works, while the transitional phases of the dynamics are fully addressed, the structure of diffusion is neglected and the relations among countries are limited to a generic analysis in which the intensity of the mutual dependence is usually represented by a defined proportion of a specific country variable on the total available for all the countries considered. Differently, in the former class of models all exchanges of inventions are fully specified. However, in order to allow for a tractable problem the analysis is confined exclusively to the steady state. This view is limiting especially from an empirical side because, though correct in principle, it drives to the possibility to estimate a dynamical problem, as it is a growth one, with only cross-sectional data as in Eaton and Kortum (1996, 1999). Neither the Dynamical Stochastic General Equilibrium approach, as in Holden T. (2010) or in The Anh P. (2007), applied to growth and innovation succeeds in dealing with both aspects since the transition phase is discarded *a priori* and also the diffusion process is not implemented through appropriate functions referred to countries. Still, the estimation widely relies on the calibration of many parameters conferring a great degree of arbitrariness to the empirical analysis. Another –agnostic- approach, like that a là Keller (2002), is only grossly linked to a theory, letting the data speak on the basis of a single equation. Again -and it could not be differently- the diffusion aspect is just referred to broad definitional categories like proximity, languages etc and not to countries interaction.

We reckon that both the structure of the diffusion, consisting in the exchanges of innovations through countries interactions, and the dynamics are fundamental to assess on growth, given the intrinsically dynamical nature of the problem. We appropriately account for this aim and developed a methodology capable of answering the question of how the process of innovation of one country is affected by all other countries. But, even more, this propriety is reflected also on the other endogenous variables. This means that, in terms of growth assessment, we are capable to discern the contribution to growth for each specific variable deriving from each country. Such a result has been performed recurring to a continuous time analysis applied to several countries whose econometric counterpart is that of continuous-time panel-data. The advantage of such an approach, in solving the above described dilemma, resides in the strict connection between the theoretical and the econometric analysis in that the latter applies straightly to the theoretical -in our case- nonlinear model under consideration. This is due to the lucky circumstance that the dynamics of growth is "naturally" expressed in continuous time. Then, we first start from a set of disequilibrium equations which allows to define a nonlinear differential system. After having studied such a model we infer on the steady state which, possibly, may also not exist. The equilibrium condition therefore is an eventuality and furthermore, even if its existence may be proved, its attainability may be complex and then to be dealt with into deep. About that, following Schumpeter (1934) first and then - among others - Nelson and Winter (1982) on the evolution of the dynamic systems with endogenous technology, we are agnostic a priori on the viability of the steady state and focus on the forces that drives the economy in disequilibrium. Moreover the approximation around the steady state, and also the evolution of the system, depends on time and countries, that involves further qualifications to understand the feasibility of the equilibrium.

Another aspect which is always missed in the analysis of growth and development with diffusion is the consideration of the effect not only of the innovation activity on the production -and vice-versabut also of its stock, which is crucial in describing the structure of the economy. In order to circumvent such an aspect, the above mentioned literature resorts to a production function based only on intermediates. Actually, the stock of technology is derived from the past and present contributions of the flow of innovations coming from all countries, which brings about a higher degree of nonlinearity in the presence of non linear behavioral functions. We correctly consider the stock of technology in the production process and account also for the connection between these two variables which is represented in our model by those business services with an intense level of knowledge (KIBS), both domestic and imported as argumented by Rubalcaba and Kox (2007). These are treated endogenously and allow us to infer on the offshoring process and its effectiveness. Moreover, the simultaneity of all the mentioned variables gains complete sense in the explanation of their interaction. In particular, knowledge intensive business services are fairly characterized by technology for the peculiarities they need in order for them to be applied and, in their turn, contribute to create new technology in the innovation sector as a consequence of their degree of specialization.

From the policy implication point of view, we are much concerned in the evaluation of how to determine in a certain future period of time a desired –and planned- outcome for a certain variable and how to control its path in order to obtain such a result. We do this by computing numerically the solution of our system and obtaining the initial conditions coherent with our targets. Further, we compute the derivative of the eigenvalues system with respect to the structural parameters of the model in order to check the changes in the stability conditions that may come from possible policy actions. Specifically, our attention is focused on the eigenvalues associated with technology, and we show a simple index capable to represent the *effort*, contributing to the dynamics, of new inventions.

Our main distinguishing features are then: a) the definition of a methodology that accounts for the two critical aspects afore mentioned (dynamics and structure); b) the implementation of a continuous time nonlinear estimation in an *exact* way, i. e. we estimate the solution of the differential system without approximating the continuous model to a discrete one; c) the treatment – with a complete dynamical analysis- of the interactions among countries not only for the innovation process, but also for the other variables and, mostly, to find out the specific country contribution, in terms of each variable, to the growth path of any other country variable; d) the sensitivity analysis of the eigenvalues (and eigenvectors) of the state-space system performed on the linearized model for all variables and for all countries; e) the evaluation of the convolution integral of the system in order to exploit the initial conditions and to obtain a desired path for the variables of interest; f) the capability to deal with the nonlinearity deriving from the introduction of the identity equation of technology.

Our work is organized as follows. In the second section we present the relations and the logic of the model. In the third section we comment on the econometric approach and on the estimated model. In the fourth section we present the results on the stability and sensitivity analysis. In the fifth section we perform the policy analysis. The sixth section concludes.

2. A key trinomial: output, technology and business services

2.1. Conceptual framework

The logic of the model rests upon the basic concept that output grows endogenously thanks to technology but, in order to link effectively these two variables, related knowledge intensive business services (we simply refer to business services for brevity) are required. This is because technology goes hand in hand with business services for firms in order to be exploited. Still, this trinomial expresses a mutual interaction since, at the same time, services are determined by output, as usual, and technology so as to be more competitive and usable. Technology in its turn depends on output and business services for the amount requested and for its implementation. All other variables are exogenous.

We consider two models, a full and a minimal one. In the former one there are several additional exogenous variables and an endogenous one, the imported services. The comparison of these two models sheds lights to understand the importance of the offshoring activity in the convergence process. The full model was estimated first by Maggi et al. (2009)¹ but without addressing this issue and the reciprocal interactions among country variables. We start describing the full model in that it comprises the minimal one.

As usual in continuous time architecture, the variables of interest adjust themselves to their relative partial equilibrium functions which depend on the associated determinants. This means that each variable is characterized by some driving forces which may not necessarily satisfy its actual value. The driving forces of output (Y) are the basic stocks of the production function: capital (K), labor (L) and technology (T); in addition there are some peculiar variables: skilled labor (HK), domestic (Sh) and imported services (Sm). The driving forces of business services (both imported and exported) are: technology, output, the intensity of the use of services in the manufacturing sector deduced from I/O tables (STR), which represents the structure of the economy and the level of regulation (REG). For technology the description of the dynamics is complicated by the fact that we have combined the flow of new inventions with the deriving stock. The inventions are measured by the count of the patent citations (Pat) from the producer country to the receiving one. This means that new inventions of a country may be produced autonomously or acquired from abroad, determining, as a whole, the total change in technology. The innovation process is therefore a bilateral one and, by definition, accounts for the interactions among countries in such a respect, i.e. there will be an equation for any country from which inventions may be acquired. It depends on the human capital (skilled) of the receiving country (HKR) and of the sender country (HKS) that uses and produces inventions respectively, other than output and business services (Sh and Sm) as explained above. Another basic bilateral variable that defines the flow of innovation is the distance (dist) between two countries whose importance is expected to decrease over time (t). We measure it with a second order effect (t*dist). Actually also HKS is a bilateral variable because the sender country may change with respect to the receiving one. Such variables characterize the model for two reasons: first they make possible the interactions among countries, secondly, though constant over time, allow for a panel estimation. These two variables are strategic for the country characterization of the diffusion process in that, by definition, it occurs in a bilateral way. The stock of technology of one country is defined as the integral over time of the summation through countries of the innovations flows. Therefore, by construction, also for one country technology may be considered for the part imputed to another country and, as a consequence, the same applies to the other endogenous variables. In such a way we are capable to discern per each country endogenous variable the contribution to its formation and dynamics deriving from the other countries. We expect that all the explanatory variables considered exert a positive effect on the dependent variable but the regulation in the business services equations and distance in the patents equations.

An attempt to obtain this characterization is to be found in Coe and Helpmann (1995), Coe et al. (2008) and Lichtenberg and van Pottelsberghe de la Potterie (1998) in which the countries

¹ The model is also referred to as the SETI where the acronym stands for Sustainable Economy development based on Technology and Innovation. A first version was estimated in the recent past by Maggi B. and was the central part of a European Commission research project and later, with new advancements, the focus of the present project.

interrelationships of technology are proxied by the bilateral imports drawn from the Trade-database IMF-Direction. Given the different focus, underlying the imports data, with respect to the core variable of such studies (traditionally R&D or patents) we reckon such a device a rough solution to the evaluation of technology diffusion, which might be biased by patterns reflecting different problems. Neither the agnostic approach of Keller (2002) seems to help in that the omission of any structural scheme -implied by the single equation adopted- does not allow for a satisfactory analysis in terms of hypotheses testing and policy implication².

2.2. The model

From the previous section we are left with the description of a nonlinear differential system in the mentioned variables referred to each country j, and accounting for the effects coming from each country i, of the following general form (Wymer C. (1997)):

(1)
$$DY_{ji}(t) = f_{ji} | Y_{ji}(t), \Theta_{ji} | + D\zeta_{ji}(t), j, i=1...n+v$$

where D is the first derivative operator and n and v represent the European and foreign countries respectively. Nine European countries are considered in the analysis: Austria, Germany, Denmark, Finland, France, the United Kingdom, Italy, Netherland, Sweden. Foreign countries are the United States and Japan. We consider eleven years during the pre-Union period 1988-1998 (annual data) to investigate on the solidity at the basis of the EU integration process. In fact the persistence of an uncertain European growth path might have been rooted before the joining of the Union, with particular reference to a not complete and appropriate exploitation of the new technology acquisitions of that period. We reckon that much responsibility for such a gap is due to the lack of an appropriate business services policy and, particularly, from the point of view of a greater openness towards an off-shoring process. Then, system (1) comprises 165 equations for any endogenous variable³ and countries. As afore mentioned, the form of the differential system is that of the partial adjustment, and the nonlinearity of our model is due to the coexistence of a definitional equation of technology, expressed in original form, and the log-transformation of the variables in the other equations. The partial equilibrium functions are indicated with the exponent pe. They are short term behavioural equations on which the disequilibrium and then the evolution of the system depend. For simplicity of notation we omit in the following system (2) the error terms which will be commented later on:

² Indeed, very few special cases for a single equation estimation are admissible (Hamilton (1994)).

 $^{^{3}}$ In total we have 15 kinds of endogenous variables, comprising the definition of technology and 11 relationships for the patenting processes.

$$\begin{cases} D \ln Y_{ji} = \alpha_{j} \left(\ln Y_{ji}^{pe} - \ln Y_{ji} \right) \\ \ln Y_{ji}^{pe} = \alpha_{j}^{0} + \alpha_{ji}^{1} \ln T_{ji} + \alpha_{ji}^{2sh} \ln sh_{ji} + \alpha_{ji}^{2sm} \ln sm_{ji} + \alpha_{ji}^{3} \ln K_{j} + \alpha_{ji}^{4} \ln L_{j} \\ D \ln Sh_{ji} = \gamma_{j}^{sh} \left(\ln Sh_{ji}^{pe} - \ln Sh_{ji} \right) \\ \ln Sh_{ji}^{pe} = \gamma_{j}^{0sh} + \gamma_{j}^{1sh} \ln Y_{ji} + \gamma_{ji}^{2sh} \ln T_{ji} + \gamma_{ji}^{3sh} \ln STR_{j} + \gamma_{ji}^{4sh} \ln ICT_{j} + \gamma_{ji}^{5sh} \ln REG_{j} \\ O \ln Sm_{ji} = \gamma_{j}^{sm} \left(\ln Sm_{ji}^{pe} - \ln Sm_{ji} \right) \\ \ln Sm_{ji}^{pe} = \gamma_{j}^{0sm} + \gamma_{j}^{1sm} \ln Y_{ji} + \gamma_{ji}^{2sm} \ln T_{ji} + \gamma_{ji}^{3sm} \ln STR_{j} + \gamma_{ji}^{4sh} \ln ICT_{j} + \gamma_{ji}^{5sh} \ln REG_{j} \\ D \ln Sm_{ji} = \beta_{ji} \left(\ln Pat_{ji}^{pe} - \ln Sm_{ji} \right) \\ \ln Pat_{ji} = \beta_{ji} \left(\ln Pat_{ji}^{pe} - \ln Pat_{ji} \right) \\ \ln Pat_{ji}^{pe} = \beta_{ji}^{0} + \beta_{ji}^{1} (a + bt) dist_{ji} + \beta_{ji}^{2} \ln HKS_{ji} + \beta_{ji}^{3sh} \ln sh_{ji} + \beta_{ji}^{3sm} \ln sm_{ji} + \beta_{ji}^{4} \ln Y_{ji} + \beta_{ji}^{5} \ln HKR_{j} \\ DT_{ji} = Pat_{ji} \end{cases}$$

The system (2) is originally of the second order reduced to the first one by means of the identity equation, which defines -and reduces- Pat_{ij} as a first order variable. Here we represent the framework of the productive structure of the economy as one centered on innovations and related services. In fact, the leading –endogenous- elements in the production of output are services and technology which are therefore modeled accordingly. Coherently, only the bilateral exogenous variables and, consequently, all the endogenous ones are characterized by two deponents indicating the country interactions. As extensively commented in Marrewijk et al. (1997), business services may be viewed in the production process as an expression of the employment of the, say, *advanced* capital such as the ICT one. The minimal model is different from (2) for the lack of the imported business services equation and the absence of STR_j , ICT_j and REG_j as explanatory variables of services. This is crucial to test the importance of the imported business services for the convergence of the system.

3. Econometric approach

As far as the estimation of system (2) is concerned, there are no enough data available for a characterization by the *i*, *j* deponents so that the 165 equations have to collapse to 15 during this phase. However, the implementation and the use of the model may well be extended to its full potentials thanks to the exogenous bilateral variables, researches and distances, which, therefore, revel themselves as strategic. Moreover, the dynamic properties concerning the convergence are different by countries because of the nonlinearity induced by the identity constraint of technology. In fact, the nonlinearity implies a different evaluation of the state-space matrix corresponding to system (2), according to the differences in time and space. The estimation of system (2) has been performed by means of ESCONA program by Wymer C. R. (2005), for panel data in continuous time. The estimation has been carried out having as a reference the exact solution of system (2), that is we did not use any approximation to calculate the model parameters in order to fit the model with the data and followed the *exact discrete analogue* procedure for non linear models. The procedure consists of the following steps: I) solve system (2); II) find the *exact* corresponding first difference system; III) set the errors structure; IV) implement the optimization procedure to find the parameters. I) and II) are solved respectively by means of the methodology based on the

exponential matrices and the appropriate choices of the initial conditions⁴. As to point III) given that the system comprises both stock and flow variables, our solution involves a double integration through the interval δ , from which the errors will be:

(3)
$$\xi(t) = \int_{t-\delta}^{t} \int_{0}^{\delta} e^{J\left[f\left(Y_{j}(\theta);\Theta_{j}\right)\right]} d\left(\zeta\left(t-\theta\right)\right) ds$$

where the exponential matrix of functions in the integral is calculated from the Jacobian of the system (1) evaluated at time t and space j, and the variance-covariance matrix is

(4)
$$\Xi_{t} = E\left[\zeta(t)\zeta'(t)\right] \text{ with } \begin{cases} E\left[\zeta(t)\right] = \mathbf{0} \\ E\left[\zeta(t_{1}) - \zeta(t_{2})\right] \left[\zeta(t_{3}) - \zeta(t_{4})\right] = \mathbf{0}, \ \forall t_{1} > t_{2} \ge t_{3} > t_{4} \\ E\left[\zeta(t+h) - \zeta(t)\right] \left[\zeta(t+h) - \zeta(t)\right] = \Omega(h) \end{cases}$$

where $\Omega(h)$ is a matrix of constants.

The important property of residuals is that, because of the integrations adopted, it may also be generated by nongaussian disturbances Dz(t), say Brownian motion or Poisson, even if z(t) and c(t) are of that sort. This is relevant in the studies on growth models since, as it is well known, innovations are subject to random discrete jumps.

In order to construct the likelihood function for the case of m=11 countries and p=15 equations, a (m*p) matrix of m blocks, of order p is considered. Each i-th block on the main diagonal represents the error covariance matrix of the p equations of country i and the off-diagonal (i, l) matrix (also of order p) is the covariance between country i and country l. The assumption made is to allow the covariances between the error terms on the equations to be non-zero and equal in each country as well as for the elements in each (i, l) of the off-diagonal matrices for pairs of countries.

The log-likelihood function of system (2), we maximize with full information, is:

(5)
$$\ln L(\Theta, \eta) = -\frac{(n+\nu)N}{2} \ln (2\pi) - \frac{1}{2} \sum_{t=1}^{N} \ln \det \Xi_t - \frac{1}{2} \sum_{t=1}^{N} (\xi_t \Xi_t^{-1} \xi_t)$$

where h is the parameters vector of the constrained variance-covariance matrix and N is the number of observations over time.

Data on GDP, services, human capital and capital are from the OECD database⁵. Data on the bilateral exchanges of technology are from the U.S. patent office⁶. The managing of this data has

⁴ For these details see Gandolfo (1981).

⁵ More specifically, all the databases used are updated at year 2000 coherently with the estimation period, GDP is collected from the OECD Main Economic Indicators, human capital from the OECD Main Science and Technology Indicators, domestic services from the OECD STAN database and data on imported services from the OECD International Trade in Services database. Physical capital and labor are taken from the Penn World Tables. Data on ICT expenditures refer to gross fixed capital formation in Information and Communication Technologies and are taken from EUROSTAT. Distance is measured in kilometers between capitals. Given the relevance of the –knowledge intensive-business services variables we specify that they are in line with the NACE 74 classification and refer to: legal, accounting, tax consultancy, market research, auditing, opinion polling, management consultancy, architectural, engineering and technical consultancy, technical testing and analyses, advertising, other business activities (see Rinaldo

involved quite some work (almost 16 millions of records!) and a special SAS code⁷, capable to retrieve and match all the correspondences one may be interested to find in the patents data, has been developed as a part of the present research. Data on regulation are from Nicoletti et al. (2000) and are referred to product market regulation⁸. Data on the structure indicator are those developed in Guerrieri and Meliciani (2005) and are based on OECD Input/Output tables ⁹. Nominal data have been deflated at 1995 prices and homogenized in dollars by means of the PPP OECD index.

Table 1 reports the estimation of system (2) on the basis of the mentioned method:

	Explanatory variables	Parameter point estimate	asymptotic s.e.	t
α_{1}	Т	0.8020	0.0920	8.72
$lpha_{\rm 2sh}$	Sh	0.1056	0.0063	16.72
$lpha_{2 \mathrm{sm}}$	Sm	0.0790	0.0035	22.29
α3	K	0.7181	0.0264	27.18
$\alpha_{\scriptscriptstyle 4}$	L	0.6871	0.0736	9.33
α	adj. speed-Y	0.0029	0.0011	2.57
$\gamma_{1 \mathrm{sh}}$	Y	0.4919	0.0138	35.59
$\gamma_{2 \mathrm{sh}}$	Т	0.3442	0.0134	25.73
$\gamma_{ m 3sh}$	Beu	5.385	0.1636	32.91
$\gamma_{ m 4sh}$	Regulation	-0.3071	0.0094	32.54
$\gamma_{\rm 5sh}$	Structure	0.5459	0.9217	25.20

Table 1. Estimation results. Full version.

et al. (2013) and Muller and Doloreux (2009) for an accurate examination of problems connected to the construction of such a variable).

⁶ Citations may be backward or forward if referred respectively to inventions discovered in the past or, from the point of view of the cited country, in the future. This, in case of a limited time series, may cause to neglect potential citations in the initial and final part of the period in the eventuality of discrepancy between the series and, respectively, the citing or cited patent or in case of lags in recording citations. To cope with this problem we follow the method indicated by B.H. Hall, A. B. Jaffe and M. Trajtenberg (2001) where it is suggested to divide each citation by the average number of citations received by the patents of the same cohort (fixed approach).

⁷ The SAS routine has been developed and implemented by Cirelli M. and Maggi B.

⁸ Such an indicator is the result of a factorial analysis though several product market indicators over the years in the sample.

⁹ In particular, in order to measure the intensity of the business services in the production of the manufacturing sector, we consider the use of business services on total value added for each manufacturing sector and for each country.

$\gamma_{\rm 6sh}$	ICT	0.2017	0.0126	16.01
γ_{sh}	adj. speed-Sh	0.0020	0.0010	2.0
$\gamma_{\rm 1sm}$	Y	0.4670	0.0176	26.59
𝒴 Y₂sm	Т	0.5517	0.0294	18.78
γ∕3sm	Сеи	2.021	0.0949	21.30
γ∕ _{4sm}	Regulation	-0.3153	0.0126	24.94
∕∕ _{5sm}	Structure	0.4992	0.0193	25.82
$\gamma_{\rm 6sm}$	ICT	0.2168	0.0101	21.49
$\gamma_{ m sm}$	adj. speed-Sm	0.0031	0.0009	3.28
β_1	(bilateral) Diffusion	0.0136	0.0057	16.16
α	Distance	-0.0213	0.0181	25.52
β	Time	0.9570	0.0064	57.93
β_2	HKS	0.5351	0.0239	22.36
$eta_{\scriptscriptstyle 3 sh}$	Sh	0.0921	0.0156	32.54
$eta_{3 m sm}$	Sm	0.4612	0.0012	10.93
β_4	Y	0.3713	0.0016	13.56
β_{5}	HKR	0.5073	0.0268	35.73
β	adj. speed-Pat _{ij}	0.0105	0.0009	11.16

As my be easily checked, all coefficients are significant and of correct sign¹⁰. We underline that the sum of the coefficients that accumulate in the production process is greater than 1 enabling, therefore, an endogenous growth process. Further, the business services equations are almost equal as expected but the coefficient for technology and the speed of adjustment, which are much greater in the case of imported business services. This is a clear indication that foreign business services may compete with respect to domestic ones thanks to the innovation process that compensates the higher costs (not explicit in the model) associated to the import activity. In fact, due to such costs, one would have expected a smaller elasticity to technology and a slower adjustment for foreign business services in case of similar levels of performance while here this is even higher than that of

¹⁰ Beu and Ceu are the constants representing the common effects in Europe for domestic and foreign business services respecively.

domestic ones to signify that the major costs of the former are more than compensated by gains in competitiveness of the latter. An additional explanation of the growing foreign business services, with the relative offshoring process, is in the presence of the same ICT, among the explanatory variables, of the domestic business services: considering that in the estimation phase the difference between the two equations is in the dependent variable, we may reasonably asses that there is a contribution for the higher speed of adjustment of the latter due to the development of ICT of the receiving country. We interpret such a result as the confirmation of what highlighted in the study on the OECD offshoring patterns (van Welsum and Vickery (2005)) where a descriptive analysis suggests and encourages to test the effect connected to ICT of the offshoring services adjustment process¹¹. Moreover, given that the largest speed of adjustment is that of technology, and in such an equation the coefficient for imported business services is almost the five-hold of the domestic ones, we reckon that such facts point out a relevant contribution to the adjustment and convergence process to be attributed to an offshoring process: on the one hand foreign business services need technology to be implemented and usable abroad, on the other hand technology is much more affected by foreign business services for their -in general- higher quality. On this point two considerations have to be done. First, there is a pervasive sluggishness in the system because of the very small speeds of adjustment, in fact they represent (see for the demonstration Gandolfo (1981)) the time required to fill the 63% of the gap between the actual and the partial equilibrium value of the variable under consideration. Second, the speeds of adjustment, if positive, are only a necessary condition for the convergence and the stability, which are not obtained as a consequence. We will perform an eigenvalue analysis to better investigate to this purpose. However, the virtuous cycle now mentioned is certainly worthy to deserve major attention. To be confirmed of that we need more statistical analysis. In particular, if our conjecture is correct, the omission of foreign business services would probably lower the speeds of adjustment. But, to consider also the possibility that the low speeds of convergence might depend on the large number of explicative variables, as this is very often the case in continuous time (see Gandolfo (1993))¹², we eliminate some exogenous variables such as ICT, REG and STR. Table 2 shows that, in this second minimal case, the speeds of adjustments are much lower than before becoming practically null in some cases as for the technology equation. Here we adopted a calibration procedure for the speed of adjustment, β , which has been interrupted at the first significant result of the parameters' t-statistics, thus confirming even more our conclusion. Neither it has been helpful to drop the mentioned variables in order to increase the speed for the domestic business services which remains almost the same.

Table 2 Estimation	results.	Minimal	version.
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Variable	Parameter point estimate	asymptotic s.e.	t
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¹¹ Arguably, in light of the globalization process, the natural step beyond in such a field of research is to endogenize ICT with respect business services themselves and human capital so as to control, in the adjustment process, for both the effects of feed-back and on the quality and the level of employment.

 $^{^{12}}$ The intuition is that the speeds of adjustment are on the main diagonal of the dynamic matrix, **A**, bringing about, because of that, an individual contribution to the rates of growth of the complete general solution as much small as greater is the number of the other coefficients to be estimated.

α_{1}	Т	0.759103	0.004811	157.78
α_{2}	S	0.687032	0.004155	165.35
α_{3}	K	0.701997	9.89E-05	7100.69
$\alpha_{\scriptscriptstyle 4}$	L	0.528219	0.003564	148.21
α	(speed of adj.)	0.000258	0.000125	2.07
γ_1	Y	0.317547	0.003625	87.6
γ₂	Т	0.708908	0.005441	130.29
γ	(speed of adj.)	0.001889	0.000162	11.66
β_1	diffusion	0.015208	0.000102	149.21
β_2	S	0.100112	0.000608	164.61
β_3	Y	0.378942	0.001854	204.4
β_4	НК	0.777034	0.004769	162.95
а	distance	-0.02002	0.00028	71.64
b	time	0.994761	0.006454	154.13
β	(speed of adj.)	0.00005	calibrated	

We therefore conclude, from the econometric approach, that the key trinomial is actually operating and, inside this, the offshoring activity of business services induces a peculiar virtuous process with the flow of technology¹³. We also observe that distance doesn't play a constant role with a negative decreasing effect over time.

4. Stability and sensitivity analysis

4.1. Countries' dynamics

We now perform a stability and sensitivity analysis, based on the full model of Table 1, to understand the relevance of the nonlinearity and the indications for economic policy purposes deriving also from the nonlinearity itself. The first thing to do is to obtain the state-space matrix, that will be, after suitable linearization, of such a form

(6) $D\mathbf{x} = \mathbf{A}\mathbf{x}$.

¹³ In Maggi and Muro (2012) the offshoring activity is evaluated also with reference to the results obtained for the steady state.

We account for the nonlinearity by considering a block diagonal matrix form with one block per each country. The nonlinearity in fact implies that for any country and any time we may observe at least –as it is the case here- different blocks in which the differences are relative to the nonlinear part of the original system:

$$\mathbf{x} = \left\{ \mathbf{x}_{j} \right\}$$
(7)
$$\mathbf{A} = \begin{bmatrix} & & \\ & \mathbf{A}_{j} \\ & & \\ & & & & \\ & & & \\ & & & \\ & & & & \\ & & & \\ & & & \\ & & & \\ & & & & \\$$

The endogenous variables and the typical block of A are:

(8)

$\mathbf{X}_{j} = \Big\{$	$\ln Y_j \ln Sh_j$	ln Sm _j lr	$n Pat_j^{AU}$	$\ln Pat_j^{GE}$	$\ln Pat_j^{DE}$	$\ln Pat_j^F$	^T In F	Pat_j^{FR} 1	n Pat $_{j}^{UK}$	$\ln Pat_j^{IT}$	ln Pa	$t_j^{JA} \ln B$	Pat_j^{NE}	$\ln Pat_j^{SW}$	$\ln Pat_{j}^{US}$	$\ln T_j \bigg\}^T$
	-α	$\alpha \alpha_2^{sh}$	$\alpha \alpha_2^{sm}$	0	0	0	0	0	0	0	0	0	0	0	ααι	
	$\gamma_1^{sh}\gamma_{sh}$	$-\gamma_{sh}$	0	0	0	0	0	0	0	0	0	0	0	0	$\gamma_2^{sh}\gamma_{sh}$	
	$\gamma_1^{sm} \gamma_{sm}$	0	$-\gamma_{sm}$	0	0	0	0	0	0	0	0	0	0	0	$\gamma_1^{sm}\gamma_{sm}$	
	$\beta^{AU}\beta_4^{AU}$	$\beta^{AU}\beta^{sh}_3$	$\beta^{AU}\beta_3^{sn}$	$\beta^{i} - \beta^{AU}$	0	0	0	0	0	0	0	0	0	0	0	
	$\beta^{GE}\beta_4^{GE}$	$\beta^{GE}\beta^{sh}_3$	$\beta^{GE}\beta_3^{sm}$	¹ 0	$-\beta^{GE}$	0	0	0	0	0	0	0	0	0	0	
	$\beta^{DE}\beta_4^{DE}$	$\beta^{DE}\beta_3^{sh}$	$\beta^{DE}\beta_3^{sm}$	¹ 0	0	$-\beta^{DE}$	0	0	0	0	0	0	0	0	0	
	$\beta^{FI}\beta_4^{FI}$	$\beta^{FI}\beta_3^{sh}$	$\beta^{FI}\beta_3^{sm}$	0	0	0	$-\beta^{FI}$	0	0	0	0	0	0	0	0	
$\mathbf{A}_{i} =$	$\beta^{FR}\beta_4^{FR}$	$\beta^{FR}\beta_3^{sh}$	$\beta^{FR}\beta_3^{sm}$	0	0	0	0	$-\beta^{FR}$	0	0	0	0	0	0	0	
J	$\beta^{UK}\beta_4^{UK}$	$\beta^{UK}\beta^{sh}_3$	$\beta^{UK}\beta_3^{sm}$	¹ 0	0	0	0	0	$-\beta^{UK}$	0	0	0	0	0	0	
	$\beta^{IT}\beta^{IT}_4$	$\beta^{IT}\beta^{sh}_3$	$\beta^{IT}\beta_3^{sm}$	0	0	0	0	0	0	$-\beta^{IT}$	0	0	0	0	0	
	$\beta^{JA}\beta_4^{JA}$	$\beta^{JA}\beta_3^{sh}$	$\beta^{JA}\beta_3^{sm}$	0	0	0	0	0	0	0	$-\beta^{J\!A}$	0	0	0	0	
	$\beta^{NE}\beta_4^{NE}$	$\beta^{NE}\beta_3^{sh}$	$\beta^{NE}\beta_3^{sm}$	¹ 0	0	0	0	0	0	0	0	$-\beta^{NE}$	0	0	0	
	$\beta^{SW}\beta_4^{SW}$	$\beta^{SW}\beta_3^{sh}$	$\beta^{SW}\beta_3^{sn}$	¹ 0	0	0	0	0	0	0	0	0	$-\beta^{SW}$	0	0	
	$\beta^{US}\beta_4^{US}$	$\beta^{US}\beta^{sh}_3$	$\beta^{US}\beta_3^{sm}$	0	0	0	0	0	0	0	0	0	0	$-\beta^{US}$	0	
	0	0	0	IC_j^{AU}	IC_j^{GE}	IC_j^{DE}	IC_j^{FI}	IC_j^{FR}	IC_j^{UK}	IC_j^{IT}	IC_j^{JA}	IC_j^{NE}	IC_j^{SW}	IC_j^{US}	$-\sum_{i=1}^{n+\nu} IC_j^i$	

Where the last row, representing the identity constraint, is affected by the point of approximation, and for this reason the acronym (*IC*) used in that entries stands for "initial condition". All β 's coefficients with an exponent indicating a country are from patents equations -even if they have been constrained to be equal.

Matrix A_j is quasi lower triangular so that we expect the coefficients on the main diagonal to be determinant for the dynamics of convergence and, from now, we may assess on their positive contribution given their positive value as from Table 1. As far as the values in the last row are concerned, they have been calculated by transforming the variables in the identity equation in logarithms and linearizing.

In fact,

$$dT_j = \sum_{i=1}^{n+\nu} Pat_{ij}$$

from which, dividing the equation for T_j and exploiting the properties of the log derivative

$$\frac{dT_j}{T_j} = \sum_{i=1}^{n+\nu} \frac{Pat_{ij}}{T_j}$$

or

$$\frac{d\ln T_j}{dt} = \sum_{i=1}^{n+\nu} e^{\ln \frac{Pat_{ij}}{T_j}},$$

which may be linearized using the Taylor series about the initial condition denoted by 0

$$\frac{d\ln T_{j}}{dt} = \sum_{i=1}^{n+\nu} \left[e^{\ln \frac{Pat_{ij}}{T_{j}}} \right|_{0} + e^{\ln \frac{Pat_{ij}}{T_{j}}} \left|_{0} (\ln Pat_{ij} - \ln Pat_{ij}) - e^{\ln \frac{Pat_{ij}}{T_{j}}} \right|_{0} (\ln T_{j} - \ln T_{j}) \right]$$

and, considering only the perturbative terms, we get

(9)
$$\frac{d\ln T_j}{dt} = \sum_{i=1}^{n+\nu} e^{\ln \frac{Pat_{ij}}{T_j}} \left(\ln Pat_{ij} - \ln T_j \right).$$

Therefore the entries in the last row will be simply the ratio between the flow of inventions from the *i*-th country to the *j*-th one upon the stock of technology of the *j*-th country, except the last entry which is referred to the total flows of inventions:

(10)
$$IC_{j}^{i} = e^{\ln \frac{Pat_{ij}}{T_{j}}}, \sum_{i=1}^{n+\nu} IC_{j}^{i}.$$

4.2 Dynamical proprieties of the model

As regards the dynamical properties of the system (6), the second element in formula (10), being the last one on the main diagonal, will be at the same time the eigenvalue that will characterize the dynamics of the several countries considered given the innovations adopted. For this reason we name it as an indicator of the *innovative effort*, that is the more a country invest in new inventions the faster approaches the steady state, provided it exists. In this connection, there are two possibilities of evaluating the steady state for this model. A first one is to consider the estimation as referred to an average European country and, for that reason, all exogenous bilateral variables have to collapse to an averaged unilateral one; which means that the concept of distance is simply referred to "abroad" in general sense and the same for the researchers. This is equivalent to say that, from a technological point of view, foreign countries are in a unique pool to which we tap irrespectively of their reciprocal interactions. Such an approach, from one side, simplifies much the analysis for the reduction of the number of the variables considered, whilst from the treatment of

the nonlinearity the difficulty increases¹⁴. We adopted a second approach where the countries specificities are accounted for in the model, and in particular consider as many stocks of technology as the associated patents flows are, over time and from any country. In such a case the difficulty of finding a closed form solution for the steady state is referred to the much larger number of variables involved and to the fact that their convergence does not imply necessarily a country convergence, being this one the result of the summation of the country variables contributions¹⁵.

Maggi and Muro (2012) addresses such an issue and, after having found a closed form solution also for this second case, elaborated a MATLAB program to study the proprieties of the steady state. The results say that the dominant rates of growth are –being dominant- pretty large coherently with the double convergence process under which the variables have to go: one ordinary and a second one due to aggregations.

This said, we can assert that the steady state does exist and the study of the convergence depends on the eigenvalues of the linearized state-space matrix and on their sensitivity to the structural model parameters. We preformed such an analysis using CONTINES program by Wymer C. (2005). Here below in Table 3 we report the eigenvalues for all countries. From the 1-th to 14-th they are almost equal through countries, admitting some small roundings, while the 15-th is country specific¹⁶. It easy to check that it identifies with the last element in the main diagonal and therefore with the initial condition of the rate of change for technology. Moreover, from the eigenvectors analysis the relevant element, in the general complete solution of the technology dynamics, is the one referred to this eigengalue. Unfortunately there is not the same clear cut for the first three eigenvalues being equally relevant, in the general complete dynamic solution, for output and services both imported and exported. It is also observable the correspondence between the speeds of adjustments of the patents equations and the eigenvalues even if only for ten of them, whilst the 11-th couples with the one of the stock of technology. Several observations are to be drawn. First, we obtain all stable eigenvalues even if the first one is very close to zero¹⁷. Therefore the model is stable and the initial conditions we used for the approximation (steady state) may be considered also as equilibrium conditions. Second, we are not assessing on the significance of the eigenvalues because of the nonlinearity of the model. In fact, given the relationship between the state-space matrix and the

there will be a country variable convergence. In fact, the result will be $z_{\bar{k}} \to 1, z_{k\neq\bar{k}} \to 0$ and so $\frac{Z}{Z} \to z_{\bar{k}}$.

¹⁶ Detailed tables for each countries available upon request.

¹⁴ In fact, in this case the identity constraint would impose that technology depends on the summation –through all countries- of the patents in natural numbers which on its turn -from the patents behavioral equations- depends on the log of technology. Such a difficulty has been overcome in Maggi et al. (2009) where a closed form solution has been found.

¹⁵ This means that if we consider a variable Z for an hypothetical country composed of K parts (k = 1,..., K) with the following dynamics per each: $Z_{k,t} = Z_1 e^{z_1 t} \dots Z_K e^{z_K t}$, the Z rate of growth (r.o.g.) will be

 $[\]frac{Z}{Z} = \frac{\sum_{k=1}^{K} z_k Z_k e^{z_k t}}{\sum_{k=1}^{K} Z_k e^{z_k t}} = \sum_{k=1}^{K} z_k p_k$, where p_k is the share of the k-th component. If $z_{\bar{k}}$ is the dominant r.o.g. only in the limit

¹⁷ In Maggi et. al. (2009), under a different context as explained before, such eigenvalue resulted close to zero and positive.

eigenvalues it is always possible (see Wymer (2005) manuals and Gandolfo (1981)) to construct a ttest for the eigenvalues but in our case a new estimation of the linearized model would have been furnished different coefficients falsifying the result. Third, the full consideration of the diffusion process in terms of an explicit interaction among countries confers the speeds of adjustments of each country the nature of the eigenvalues, which therefore become crucial for the attainment to the equilibrium. Fourth and importantly, the technology eigenvalue has been found to be dominant, being the highest in absolute value, and therefore the most relevant for growth and stability. This is the confirmation of the same result obtained in the literature with other structural approaches (see for instance Eaton and Kortum (1999)). From a pure conceptual point of view, here the rate of change of technology represents the fuel of the productive process assisted by business services and, for such a reason, is the main eigenvalue. The relevance for the associated index, afore mentioned, is in such an explanation. Its range is [0, 1] and is decreasing over time, by fixing theoretically that at the beginning of the observation period the change equals the stock. Such a property gives, in its simplicity, the possibility to make comparisons at parity conditions, between different countries, on the effort they are *currently* undertaking in the stability and convergence process, where the adjective currently is to emphasize the effect of nonlinearity which modifies the state-space matrix at any instant. Therefore, what really matters in the nonlinear dynamics is the capability of the current conditions to settle the bases for the future speed of convergence, which is, as time passes, what is represented by the dominant eigenvalue under consideration. At this purpose we observe that the fastest convergence process to the equilibrium is attributable to Germany followed by Japan and US after which are the other countries in a, more or less, homogeneous way with the exception of Sweden which figures as the last one. Actually, as it is shown in Maggi and Muro (2012), the path of this critical eigenvalue was in the past better for the US and for Sweden in Europe. From a technical point of view, this is an implication of that matrix A_i in formula (8) is time and space varying. Accordingly, for each country the last eigenvalue has been calculated as the time average of the contribution from all countries to the relative change in the stock of technology. Therefore, in this study, the economic counterpart of the nonlinearity is that each country may modify the eigenvalues at each time as happened for Sweden which undertook the highest investment in ideas at the beginning of the sample period, and consequently the highest eigenvalue at that time, but not so in the final part. It goes without saying that such arguments are important for the analysis of the convergence to and the stability of the steady state for which what is relevant are the coefficients of the endogenous variables in the homogeneous equations of which the eigenvalues are complex function¹⁸. Differently, in the analysis of the steady state what matters are the rates of growth which are clearly linked to the path of the endogenous and exogenous variables. In such an analysis the ranking of countries for the rates of growth of technology may well be different from that of Table 3, as found in Maggi and Muro (2012) where Sweden jumps at the first place. They have been found supported by the almost-highest rates of growth of business services, especially if imported, which on their turn are linked to a consistent rate of growth of ICT. Of course, the initial levels of endogenous variables account for all past investments in innovations.

¹⁸ As anticipated before, we consider λ_{15} as the "eigenvalue of technology" we have in mind that in the general complete solution of homogeneous system associated to (2) the dynamics of technology is characterized by very small eigenvectors elements associated to the first 14 eigenvalues and a significant one to the 15°.

Table 3. Stability Analysis.

Eigenvalues	Real part	Modulus	Damping period
λ_1	-0.00014	0.00014	6945.978
λ ₂	-0.00215	0.00215	465.978
λ_3	-0.00391	0.00391	255.606
λ_4	-0.0105	0.0105	95.238
λ_5	-0.0105	0.0105	95.238
λ_6	-0.0105	0.0105	95.238
λ_7	-0.0105	0.0105	95.238
λ_8	-0.0105	0.0105	95.238
λ_9	-0.0105	0.0105	95.238
λ_{10}	-0.0105	0.0105	95.238
λ_{11}	-0.0105	0.0105	95.238
λ ₁₂	-0.0105	0.0105	95.238
λ_{13}	-0.0105	0.0105	95.238
λ_{14}	-0.01220	0.01225	81.664
Austria: $-\sum_{i=1}^{n+\nu} IC_{AU}^i = \lambda_{15}^{AU}$	-0.15888	0.15888	6.294
Germany: $-\sum_{i=1}^{n+\nu} IC_{GE}^i = \lambda_{15}^{GE}$	-0.49056	0.49056	2.038
Denmark: $-\sum_{i=1}^{n+\nu} IC_{DE}^{i} = \lambda_{15}^{DE}$	-0.16848	0.16848	5.935
Finland: $-\sum_{i=1}^{n+\nu} IC_{FI}^i = \lambda_{15}^{FI}$	-0.17299	0.17299	5.781
France: $-\sum_{i=1}^{n+\nu} IC_{FR}^{i} = \lambda_{15}^{FR}$	-0.15167	0.15167	6.593
$UK: -\sum_{i=1}^{n+\nu} IC_{UK}^i = \lambda_{15}^{UK}$	-0.15287	0.15287	6.541
Italy: $-\sum_{i=1}^{n+\nu} \overline{IC_{IT}^{i}} = \lambda_{15}^{T}$	-0.15477	0.15477	6.461

Japan: $-\sum_{i=1}^{n+\nu} IC_{JP}^{i} = \lambda_{15}^{JP}$	-0.31354	0.31354	3.189
Netherland: $-\sum_{i=1}^{N+\nu} iC_{NE}^{i} = \lambda_{15}^{NE}$	-0.15167	0.15167	6.593
Sweden: $-\sum_{i=1}^{n+\nu} IC_{SW}^i = \lambda_{15}^{SW}$	-0.12174	0.12174	8.215
$US: -\sum_{i=1}^{n+\nu} IC_{US}^i = \lambda_{15}^{US}$	-0.1943	0.1943	5.147

The small speeds of adjustment are coherent with the high dumping period observed and confirm the difficulty for Europe to approach the stability even if the effect of the dominant technology eigenvalue tends to reduce this problem. This suggests to find, possibly, some other explanatory variables, concerning the functioning of the institutions or the social organization, in order to understand the present sluggishness.

As for the sensitivity, we evaluate the impact, on the convergence and stability, of a change in the structural parameters. This analysis moves from the basic relationship between eigenvalues and eigenvectors and exploit the following formulas to answer the now mentioned question:

(11)
$$\frac{\partial \lambda_{ji}}{\partial A_{j}} = \left[\frac{\partial \lambda_{ji}}{\partial a_{jik}}\right] = h_{ji}^{*}h_{jk}^{'}$$

(12)
$$\frac{\partial \lambda_{ji}}{\partial g_{jl}} = \sum_{i} \sum_{k} \frac{\partial \lambda_{ji}}{\partial a_{jik}} \frac{\partial a_{jik}}{\partial g_{jl}} ; j=1...n+v; i, k: 1...15.$$

Where \mathbf{A}_j is the state-space matrix with generic element a_{jik} , \mathcal{G}_l is the *l*-th structural parameter of an endogenous variable of system (2), λ_i is the *i*-th eigenvalue, h_i^* the *i*-th transposed row vector of the inverse eigenvector matrix and h_k the *k*-th transposed column vector of the eigenvector matrix (a detailed proof is in Gandolfo (1981)). The implementation of formulae (11) and (12) brings to the elaboration of n+v = 11 sensitivity matrices, as many as the countries considered but, given the strong similarities of the outcoming figures we concentrate our results uniquely in Table 4¹⁹. Such similarities reside in the fact that, as said, our estimation is country specific thanks to the presence of bilateral variables and of some dummy constant variables but not for the characterizations of the coefficients in formula (12) apart those of the linearized equation. For the same argument in Table 4 the impact on the eigenvalues of IC_j^i is the same as that of T^j , being the former an additional part of the latter. A straightforward, but nonetheless relevant, result is the 100% -favourable- impact of the last element in the main diagonal of \mathbf{A}_j on the same 15-th eigenvalue, and of the speed of adjustment²⁰ of the innovations processes on the eigenvalues numbered from 4 to 13. However it is

¹⁹ The whole set of tables is of course available upon request.

²⁰ Note the sign in the sensitivity matrix is the opposite coherently with formula (9).

valuable noticing that all speeds of adjustment exert a generalized beneficial effect to the stability and convergence, meaning that the partial adjustment relationships are worthy to be encouraged in reaching the targets. This may be typically done by reducing the costs of bureaucracy, in terms of binding and protecting legislation in market (not primary) activities and, more in general, the cost of politics, as far as the political decisional levels are concerned. Also the coefficients of the other variables *Y*, *Sh*, *Sm* and *T* in the several equations have a strong impact but with a mixed effects, which is to be expected given the complexity of the interactions in the model. As to the comparisons among countries in terms of *innovative effort* index, a small though relevant evidence is that the sensitivity reveals a constantly stronger beneficial effect in the less performing countries, especially when referred to innovations and technology. This is, once again, in favour to invest in the research activity especially in the case of low performance.

	eigenvalues	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9	λ_{10}	λ_{11}	λ_{12}	λ_{13}	λ_{14}	λ_{15}
eq. Y	variables															
α_1	Т	0.3975	0.0197	-0.1382	0	0	0	0	0	0	0	0	0	0	-0.3055	0.0265
α2	Sh	0.361	-0.1685	-0.2056	0	0	0	0	0	0	0	0	0	0	0.0132	-0.0001
α^m_{2}	Sm	0.4443	0.058	-0.5462	0	0	0	0	0	0	0	0	0	0	0.0442	-0.0003
α	adj. speed-Y	-0.4124	-0.0114	-0.5026	0	0	0	0	0	0	0	0	0	0	-0.074	0.0004
eq. Sh																
γ1	Y	0.1446	-0.056	-0.1077	0	0	0	0	0	0	0	0	0	0	0.0191	-0.0001
γ2	Т	0.1394	-0.0968	0.0296	0	0	0	0	0	0	0	0	0	0	-0.0788	0.0066
γ	adj. speed-Sh	-0.1266	-0.826	-0.044	0	0	0	0	0	0	0	0	0	0	-0.0034	0
eq. Sm																
γ^m_{1}	Y	0.2857	0.0303	-0.4317	0	0	0	0	0	0	0	0	0	0	0.1162	-0.0005
γ^m_2	Т	0.2753	0.0524	0.1187	0	0	0	0	0	0	0	0	0	0	-0.4799	0.0334
γ ^m	adj. speed-Sm	-0.3078	-0.1539	-0.4692	0	0	0	0	0	0	0	0	0	0	-0.0694	0.0004
Eq. Pat _{AUj}																
β_1	Sh	0.1307	-0.0717	-0.0232	0	0	0	0	0	0	0	0	0	0	-0.0404	0.0046
β_2	Sm	0.1609	0.0247	-0.0616	0	0	0	0	0	0	0	0	0	0	-0.1355	0.0116
β_3	Y	0.1493	0.0049	0.0566	0	0	0	0	0	0	0	0	0	0	-0.2268	0.016
β	adj. speed-Pat _{ij}	-0.1438	-0.0083	0.0152	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-0.864	0.0008
Eq. T																

Table 4. Sensitivity analysis.

IC_{j}^{AU}	Pat_{j}^{AU}	0.0094	0.0004	-0.0006	0	0	0	0	0	0	0	0	0	0	-0.0099	0.0008
IC_{j}^{GE}	Pat_{j}^{GE}	0.0031	0.0001	-0.0002	0	0	0	0	0	0	0	0	0	0	-0.0031	0.0001
IC_{j}^{DE}	Pat_{j}^{DE}	0.0088	0.0004	-0.0006	0	0	0	0	0	0	0	0	0	0	-0.0093	0.0007
IC_{j}^{FI}	Pat_{j}^{FI}	0.0086	0.0004	-0.0006	0	0	0	0	0	0	0	0	0	0	-0.0091	0.0006
IC_{j}^{FR}	Pat_{j}^{FR}	0.0098	0.0005	-0.0007	0	0	0	0	0	0	0	0	0	0	-0.0104	0.0009
IC_{j}^{UK}	Pat_{j}^{UK}	0.0097	0.0005	-0.0007	0	0	0	0	0	0	0	0	0	0	-0.0104	0.0008
IC_{j}^{IT}	Pat_{j}^{T}	0.0096	0.0004	-0.0006	0	0	0	0	0	0	0	0	0	0	-0.0102	0.0008
IC_{j}^{JP}	Pat_{j}^{JP}	0.0048	0.0002	-0.0003	0	0	0	0	0	0	0	0	0	0	-0.0049	0.0002
IC_{j}^{NE}	Pat_{j}^{NE}	0.0098	0.0005	-0.0007	0	0	0	0	0	0	0	0	0	0	-0.0105	0.0009
IC_{j}^{SW}	Pat_{j}^{SW}	0.0122	0.0006	-0.0008	0	0	0	0	0	0	0	0	0	0	-0.0133	0.0014
IC_{j}^{US}	Pat_{j}^{US}	0.0077	0.0004	-0.0005	0	0	0	0	0	0	0	0	0	0	-0.008	0.0005
$-\sum_{i=1}^{n+\nu} IC^i_{AU}$	T^{AU}	0.0094	0.0004	-0.0006	0	0	0	0	0	0	0	0	0	0	-0.0108	1.0016
$-\sum_{i=1}^{n+\nu} IC_{GE}^i$	T^{GE}	0.0031	0.0001	-0.0002	0	0	0	0	0	0	0	0	0	0	-0.0031	1.0002
$-\sum_{i=1}^{n+\nu} IC_{DE}^i$	T^{DE}	0.0088	0.0004	-0.0006	0	0	0	0	0	0	0	0	0	0	-0.0101	1.0014
$-\sum_{i=1}^{n+\nu} IC^i_{FI}$	T^{FI}	0.0086	0.0004	-0.0006	0	0	0	0	0	0	0	0	0	0	-0.0098	1.0013
$-\sum_{i=1}^{n+\nu} IC_{FR}^i$	T^{FR}	0.0098	0.0005	-0.0007	0	0	0	0	0	0	0	0	0	0	-0.0114	1.0018
$-\sum_{i=1}^{n+\nu} IC_{UK}^i$	T^{UK}	0.0097	0.0005	-0.0007	0	0	0	0	0	0	0	0	0	0	-0.0113	1.0017
$-\sum_{i=1}^{n+\nu} IC_{IT}^i$	$T^{\prime T}$	0.0096	0.0005	-0.0007	0	0	0	0	0	0	0	0	0	0	-0.0111	1.0017
$-\sum_{i=1}^{n+\nu} IC_{JP}^{i}$	T^{JP}	0.0048	0.0002	-0.0003	0	0	0	0	0	0	0	0	0	0	-0.0051	1.0004

$-\sum_{i=1}^{n+v} IC_{NE}^{i}$	T^{NE}	0.0098	0.0005	-0.0007	0	0	0	0	0	0	0	0	0	0	-0.0114	1.0018
$-\sum_{i=1}^{n+\nu} IC_{SW}^i$	T^{SW}	0.0122	0.0006	-0.0008	0	0	0	0	0	0	0	0	0	0	-0.0148	1.0029
$-\sum_{i=1}^{n+\nu} IC_{US}^i$	T^{US}	0.0077	0.0004	-0.0005	0	0	0	0	0	0	0	0	0	0	-0.0086	1.0011

We conclude this section by observing that from the analysis now developed we can only asses on the stability of the growth process towards the equilibrium. Another approach to study the dynamics, accounting also for the initial levels, is that of considering the evolution of the system under consideration.

5. Policy implications: controlling for the growth-path evolution

The presence of the exogenous variables in system (2) may affect both the rates of growth and initial levels of the endogenous variables on the basis of an *a priori* known behavior or a control rule. Maggi and Muro (2012) devoted special attention to such an issue by studying, in particular, their functional form with respect to the parameters and the control variables, that gives the opportunity to evaluate the comparative dynamics of the model. In this section we study the dynamics of our model looking at its actual path and asking whether and how it is possible to match with a target value for the endogenous variables in the future. In pursuing such a target we might also be asked to answer about when the intervention to this aim is more appropriate to occur and, possibly the length of the period required to get the desired result, so that both the initial values and the time elapsing become policy instruments. These problems may find the answer in the implementation of the convolution integral associated to system $(2)^{21}$. The first step is to start from the complete general linearized solution of such a system, that is for the generic *j*-th country at time *t*:

(13)
$$\mathbf{x}_{j}^{o}(t) = e^{\mathbf{A}_{j}(t-t_{0})} \mathbf{x}_{j}^{o}(0) + \int_{t_{0}}^{t} e^{\mathbf{A}_{j}(t-\theta)} \mathbf{B}_{j} \mathbf{u}_{j}^{o}(\theta) d\theta$$

where the \mathbf{A}_j matrix assumes the role of the Jacobian, the superscript *o* indicates the double integration because of the presence of stocks and flows coherently with formula (3), and the \mathbf{B}_j matrix is country specific since it includes the distances of all countries with respect to the *j*-th one, other than the estimates of the exogenous variables parameters. This is because, here, in order to compute the integral, we group all the constants referred to the control –exogenous- variables in \mathbf{B}_j , which is therefore associated to a **B** matrix of order 165*209. The $\mathbf{u}_j(t)$ vector is composed of the following exogenous elements:

²¹ To this aim a Matlab code has been appositely written and tested.

(14)
$$ICT_{j} = ICT_{j}(0)e^{\rho_{ICT_{j}t}}; \quad K_{j} = K_{j}(0)e^{\rho_{K_{j}t}}; \quad L_{j} = L_{j}(0)e^{\rho_{L_{j}t}}; \quad HKS_{ji} = HKS_{ji}(0)e^{\rho_{HKS_{ji}t}}$$

 $HKR_{j} = HKR_{j}(0)e^{\rho_{HKR_{j}t}}; \quad REG_{j} = REG_{j}(0)e^{\rho_{REG_{j}t}}; \quad STR_{j} = STR_{j}(0)e^{\rho_{STR_{j}t}}; \quad t ; const$

where both the initial conditions and the rates of growth (ρ_j) for the variables have been calculated from the data coherently with the historical paths. The **B**_j matrix is of a shape like this:

	αα₀	$\alpha \alpha_3^K$	$\alpha \alpha_A^L$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$\gamma_{sh}\gamma_0^{sh}$	0	0	$\gamma_{sh}\gamma_{3}^{STR}$	$\gamma_{sh}\gamma_4^{ICT}$	$\gamma_{sh}\gamma_5^{REG}$	0	0	0	0	0	0	0	0	0	0	0	0	0
	$\gamma_{sm}\gamma_0^{sm}$	0	0	$\gamma_{sm}\gamma_{3}^{STR}$	$\gamma_{sm}\gamma_4^{ICT}$	$\gamma_{sm}\gamma_5^{REG}$	0	0	0	0	0	0	0	0	0	0	0	0	0
	$\beta^{AU}\left(\beta_{0}^{AU}+\beta_{1}^{AU}a^{*}dist_{j}^{AU}\right)$	0	0	0	0	0	$\beta^{AU}\beta_5^{AU,HKR}$	$\beta^{AU}\beta_{l}^{AU}b^{*}dist_{j}^{AU}$	$\beta^{AU}\beta_2^{AU,HKS}$	0	0	0	0	0	0	0	0	0	0
	$\beta^{GE} \left(\beta_0^{GE} + \beta_1^{GE} a^* dist_j^{GE} \right)$	0	0	0	0	0	$\beta^{GE}\beta_5^{GE,HKR}$	$\beta^{GE}\beta_1^{GE}b^*dist_j^{GE}$	0	$\beta^{GE}\beta_2^{GE,HKS}$	0	0	0	0	0	0	0	0	0
	$\beta^{DE}\left(\beta_{0}^{DE}+\beta_{1}^{DE}a^{*}dist_{j}^{DE}\right)$	0	0	0	0	0	$\beta^{DE}\beta_5^{DE,HKR}$	$\beta^{DE}\beta_1^{DE}b^*dist_j^{DE}$	0	0	$\beta^{DE}\beta_2^{DE,HKS}$	0	0	0	0	0	0	0	0
	$\beta^{FI} \left(\beta_0^{FI} + \beta_1^{FI} a^* dist_j^{FI} \right)$	0	0	0	0	0	$\beta^{FI}\beta_5^{FI,HKR}$	$\beta^{FI}\beta_1^{FI}b^*dist_j^{FI}$	0	0	0	$\beta^{FI}\beta_2^{FI,HKS}$	0	0	0	0	0	0	0
$\mathbf{B}_i =$	$\beta^{FR} \left(\beta_0^{FR} + \beta_1^{FR} a^* dist_j^{FR} \right)$	0	0	0	0	0	$\beta^{FR}\beta_5^{FR,HKR}$	$\beta^{FR}\beta_1^{FR}b^*dist_j^{FR}$	0	0	0	0	$\beta^{FR}\beta_2^{FR,HKS}$	0	0	0	0	0	0
,	$\beta^{UK} \left(\beta_0^{UK} + \beta_1^{UK} a^* dist_j^{UK} \right)$	0	0	0	0	0	$\beta^{UK}\beta_5^{UK,HKR}$	$\beta^{UK}\beta_1^{UK}b^*dist_j^{UK}$	0	0	0	0	0	$\beta^{UK}\beta_2^{UK,HKS}$	0	0	0	0	0
	$\beta^{IT} \left(\beta_0^{IT} + \beta_1^{IT} a^* dist_j^{IT} \right)$	0	0	0	0	0	$\beta^{IT} \beta_5^{IT,HKR}$	$\beta^{IT}\beta_1^{IT}b^*dist_i^T$	0	0	0	0	0	0	$\beta^{IT}\beta_2^{IT,HKS}$	0	0	0	0
	$\beta^{JP} \left(\beta_0^{JP} + \beta_1^{JP} a^* dist_i^{JP} \right)$	0	0	0	0	0	$\beta^{JA}\beta_5^{JA,HKR}$	$\beta^{JA}\beta_1^{JA}b^*dist_i^{JA}$	0	0	0	0	0	0	0	$\beta^{JA}\beta_{2}^{JA,HKS}$	0	0	0
	$\beta^{NE}\left(\beta_{0}^{NE}+\beta_{1}^{NE}a^{*}dist_{i}^{NE}\right)$	0	0	0	0	0	$\beta^{NE}\beta_5^{NE,HKR}$	$\beta^{NE} \beta_1^{NE} b^* dist_i^{NE}$	0	0	0	0	0	0	0	0	$\beta^{NE}\beta_{2}^{NE,HKS}$	0	0
	$\beta^{SW}(\beta_0^{SW} + \beta_1^{SW}a^*dist_i^{SW})$	0	0	0	0	0	$\beta^{SW}\beta_5^{SW,HKR}$	$\beta^{SW} \beta_1^{SW} b^* dist_i^{SW}$	0	0	0	0	0	0	0	0	0	$\beta^{SW}\beta_2^{SW,HKS}$	0
	$\beta^{US}(\beta_0^{US} + \beta_1^{US}a^*dist_i^{US})$	0	0	0	0	0	BUS BUS, HKR	$\beta^{US} \beta^{US}_{t} b^* dist^{US}_{t}$	0	0	0	0	0	0	0	0	0	0	BUS BUS, H
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From formula (13), by imposing at the final time the target value $\mathbf{x}_{j}^{o}(t_{fin})$, it is possible to retrieve the initial condition at the beginning of the desired elapsing period $\mathbf{x}_{i}^{o}(t_{in})$:

(15) $\mathbf{x}_{j}^{o}(t_{in}) = \left[e^{\mathbf{A}_{j}\left(t_{fin}-t_{in}\right)}\right]^{-1} \mathbf{x}_{j}^{o}\left(t_{fin}\right) - \int_{t_{in}}^{t_{fin}} e^{\mathbf{A}_{j}\left(t_{fin}-\theta\right)} \mathbf{B}_{j} \mathbf{u}_{j}^{o}\left(\theta\right) d\theta.$

The above formula has been implemented by remembering that $e^{A_j} = \sum_{k=0}^{+\infty} \frac{A_j^k}{k!}$ and using the trapezoidal rule with a reasonable small pace of integration (o.d.g = 110⁻¹). From this calculus we obtain that, given a final target value, it is possible to impose reasonable (i.e. compatible with the historical values) intermediate targets –initial values- of the endogenous variables in order to get for three consecutive years an increment for real output in the range (percentage values) of [1, 1.33], for business services (domestic and imported) [2, 2.5] and for patents and technology [3, 3.5]. Importantly, the same total increments would not have been viable in a different period of time or replicable by shifting the same time interval, moreover such increments may be different for the countries considered and, of course, there exists an interdependence among countries in such a respect.

We tried also several additional experiments, conducted separately, to better qualify the impact of a change in the exogenous variables on the results now commented. In particular, maintaining the same target values as before, we evaluated the results on the initial conditions coming from: 1) doubling specialized personnel (Human Capital) and 2) ICT, 3) halving Regulation. We reckon as beneficial the effects consisting in a reduction of the initial conditions of the endogenous variables within the same 3-year time-interval of the previous simulation, when improvements of policies 1)-3) were not implemented. Unfortunately, but not surprisingly, we obtained confirmation of the slow –though potentially relevant- dynamics of the estimated system in the short run. We did obtain

beneficial results but, as for policy 1) it requires a time interval of at least 5 years (i.e. almost the double of the previous experiment) for output to get a decrement in the initial condition significantly different from 0 as well as for domestic and foreign services, while technology performs, with a decrement –averaging through countries- of about 0.8%, maintaining the interval of 3 years. As for policies 2) and 3) we got very similar results for output and a decrement of about 0.03% and 0.05% for domestic and foreign services respectively while, for policy 3), they amount to 0.04% and 0.07% in the 3-years interval. The effects on new inventions are again more consistent around 0.4% for both policies in the short run. However, beyond the quantitative results recordable what emerged in qualitative terms is that the time interval of five years, i.e. the medium term, is the period required to start looking at any improvements deriving from the policies experimented, which is coherent with the sluggishness observed and pictured by the small speeds of adjustments and eigenvalues estimated. Technically, this means that the surface represented by integral (15) accelerates over time only from the medium-long run on and that, within the short run, the only ascertainable, although small, improvements occur in the non-manufacturing sectors i.e. business services and inventions. In the long run the improvements of output from the policies adopted are, as expected, remarkable and go from an-approximately- additional 1% per year in the case of doubled human capital and slightly more for the other two policies because of the longer time required for new inventions to be accumulated across countries and embedded in the production function.

Summing up, we underline the following points. First, specific initial conditions in time are required for growth to be viable and, to this aim, it is necessary an appropriate dynamic model capable to pick frictions and lags in the countries considered. Second, the adjustment required in the manufacturing sector to the innovation process is a key aspect in promoting growth. Third, in accordance with what observed in section 3, the reaction of foreign business services to the policies implemented is always higher than that of domestic ones thus revealing, again, a greater capacity of competing.

Other important studies, on the similar line, might be conducted in terms of the implications in the use of **B** and **u** with interest also in other strategic variables for the Union such as the ones representing the effect of inequality or social inclusion.

6. Conclusions and further research

This research provides an alternative method for the study of the structural models in economic dynamics. The main characteristic is that the continuous time model developed may be theoretically studied and statistically implemented by *exactly* matching its dynamics with the data. Among the several advantages, we do not impose *a priori* an equilibrium and the disequilibrium relations we use serve to study the transitional dynamics in a Schumpeterian evolutionary context. In the specific framework of growth, business services and technology, we have found their interaction significant and of important implications. Summing up, we underline: I) the role of rate of growth of technology as a stabilizer of the economy given its intrinsic nature of eigenvalue; II) the possibility to improve such a stabilizer over time and across countries, in terms of new initial conditions, according to the nonlinearity of the model; III) the relevance of the business services in this process as a vehicle of technology towards the production and with a virtuous interaction together with technology itself; besides, such a process highlights the benefits of an offshoring activity. We

characterized such results on a detailed geographical bases by estimating systems of continuous time panel data. To that aim we used explicative bilateral variables such as distance and researchers. A certainly promising research area in this field is to posing in continuous way also the space, like in Donaghy and Plotnikova (2004). Specifically, it would be possible to avoid to consider as homogeneous wide geographic areas or to test the similarities in regions inside different areas.

The small speeds of adjustment from our model confirm the difficulty for Europe to approach the stability even if preserved by the dominant eigenvalue of technology, and suggest to find some other explanatory variables, concerning the functioning of the institutions or the social organization, in order to understand the prolonged present sluggishness. To conclude, one interesting avenue for future research also relates to considering current data, in order to see how the consolidation of the EU integration process and the recent crisis may affect the dynamics of the model.

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