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(*Why*) *Is The Euro System Intrinsically Unstable*?

Bernardo Maggi*

Abstract

In this study we focus on the dynamics of taxation, debt, and monetary stability in a currency union area. We specifically adapt our theoretical set up to the Euro zone with special emphasis on the countries affected by critical conditions of public debt. We deal with such a problem in a dynamic optimization perspective by referring to the optimal control literature and find the optimal taxation and composition by maturity of the debt as it follows from the Stability and Growth Pact (SGP). Critical results depend upon the accumulation over time of the past decisions on public expenses and the consequent high level of taxation rate according to which a probability of failure to comply the SGP is evaluated.

J.E.L.: H63, H21, F40, C61

1. *Introduction*

In this paper the problem of the feasibility of the Euro system is addressed especially for countries where the level of taxation is extremely high compared to the services offered. We describe the Government behaviour of our representative country in conformity with the prescriptions imposed by the Stability and Growth Pact (SGP) agreed within the European countries on the base of the objectives and the treaties of the European Union. This pact may be synthetically outlined with the following economic prescriptions for the member states: 1) to limit the public expenditure in order to respect the threshold level of the deficit-GDP ratio and consequently that of public debt-GDP ratio; 2) to adopt an efficient program of managing taxes in order to reduce as much as possible the distortion of the economic system due the fiscal pressure and so to guarantee a competitive market; 3) to respect the constraint of the public budget without the possibility of creating monetary

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base. In this paper it is shown that such an optimal control problem drives to the result that the Euro system is not feasible in case of an uncertain decisional process of the public expenditure and of low output growth. This uncertainty on the decisional process reflects the unclearness or ambiguity of the policy makers in proposing first and realizing then a program of expenditure. Examples may be of several types such as the uncertainty on labour or pensions laws that suppress due payments later recognized unjustified by the court and then to be paid[1], or more generally the expenses unknown and unexpected to the community for several reasons due to the autonomy of the institutions or to the fact that are expenses indirectly linked to the public sector as in the case of the privatized public companies in financial difficulties which require financial support by the Government in that have strong influence on the public utility. Synthetically, the criticalities that emerge from the dynamic interpretation of the SGP pact are principally due to such an uncertainty and its influence on the future level of expenditure, which makes the system intrinsically unstable and vulnerable since the average tax rate required to finance such a system tends to become socially unsustainable in connection with the services provided by the public sector.

The aim of the present paper is to highlight that the strict rules established in the SGP for the EU members are not applicable in the long run especially for those countries which are not capable to solve kernel problems such as: a not viable tax rate, a high public debt, an uncertain public expenditure, a poor rate of growth and appropriate levels of interest rates. The practice of just re-defining with the annual budget new initial conditions of the crucial variables of the budget constraint and a coherent path of taxation under the respect of the SGP is purely a palliative treatment. Furthermore, this worsens the conditions of the public finance by exacerbating the fiscal pressure and conducts the Government to impose not sustainable levels of the average taxation rate from the social point of view, i.e. compared

[1] In this regard, an exemplary case is that one occurred in Italy where the Constitutional court recently judged in favor of refunding the interruption of both the long service bonus and the indexation of pensions for many billions of euros.

to the public services provided. The relevant question is that one cannot know how long may last such a way of managing the public finance.

A similar problem, in the context of the currency-crises linked to the public debt and the budget constraint, was addressed by the literature of the pre-UE period as a consequence of the crises of the fixed exchange rate arrangements occurred at that time (see among others Alesina et al., 1990, Obstfeld 1986, 1994). However, it returns in vogue at the present (Galina and Obstfeld, 2014) as a consequence of the systemic instability diffused through the Euro zone. Obstfeld (2013) claims that the risk of uncertain and sudden public expenses is currently even more serious in the case of Government emergency rescue due to the painfulness of banks default for the actual large size of the banks dimension in Europe compared to GDP. This is a clear example of uncertainty in the public expenditure due to institutions indirectly linked to the public sector for the undoubtedly public utility that the banking sector has even when privatized.

The paper is organized as follows. Section 2 analyse and solves the control problem implicit in the SGP, section 3 evaluates the failure probability to comply the SGP pact, section 4 considers the consequences of relaxing the rules of the SGP, section 5 studies the dynamics of the state variable represented by the total debt, section 6 explores the optimal dynamics of the debt subdivided by maturity to accommodate the refunding plan, section 7 proposes an empirical evaluation of the SGP risk failure probability, section 8 concludes and finally the Appendix will report a variant of the model proposed and some algebra underlying the main text.

2. *Theoretical set up*

The loss function, $\psi(\tau_t)$, represents the cost imputed to the public sector in terms of the inefficiencies due to the “deadweight losses" or "excess burdens", the cost of collecting taxes and, more generally, all the Government policy actions representable by fiscal receipts (Barro, 1979). For practical use of minimization $\psi(\tau_t)$ may be assumed of quadratic form (Tanner and Carey, 2005). Then, the minimization of such a function is the prescription to respect for a Government to be efficient in managing taxes.

The budget constraint is described along the following four characterizing hypotheses implied by the European SGP,

$$(1) \qquad B_t - B_{t+1} + g_t + B_t\, r = \tau_t \,.$$

In this expression the monetary base is absent since the Central banks of the single European countries cannot create money any longer (hypothesis 1). Still, price stability is assumed in accordance to the stability of money (hypothesis 2). $\delta = (1+r)^{-1}$ is the intertemporal discount factor dependent on the interest rate r, g_t is the public expenditure and the amount of interest on the public debt at time t is $r * B_t$. The interest rate is constant under the interest rates convergence assumption of the SGP (hypothesis 3) and the public expenditure is not explosive and so it is stationary (hypothesis 4) in order to take under control the deficit.[2] Then, the public expenditure goes under an AR(1) process given a certain initial condition, g_{t_0}:

$$(2) \qquad g_t = \rho\, g_{t-1} + \varepsilon_t, \ |\rho|<1, \ E_t(\varepsilon_{t+1}) = 0, \ \ \varepsilon_t \sim \text{i.i.d}, \ g_{t_0} \geq 0.$$

This is a crucial hypothesis in that will serve to show that the monetary Union, notwithstanding the respect of stringent rules on the public expenditure, is intrinsically unstable. We underline that the introduction in (2) of a fixed expenditure per each year would have complicated the analysis without altering the main conclusions. Furthermore, the consideration of a fixed expenditure would have exacerbated the budget constraint so that the result of intrinsic instability implied by the SGP would be even more valid. The Appendix will address such an issue.

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[2] The rules of the SGP pact are founded on the European Treaty. For major details see the European Treaty at articles 3 for hypothesis 1), 140 for hypothesis 2) and 3), 126 for hypothesis 4).

The public debt on which are computed the interest payments is evaluated at the beginning of each period while the public expenditure (real and financial) and the tax revenue at the end.

Formally, the dynamic minimization problem of the mentioned loss function may be solved by referring to the control theory. In fact, B_t, τ_t, g_t, may be interpreted respectively as state, control and decisional variables[3]. ρg_{t-1} is the current expenditure deriving from the contracts of the preceding period and ε_t is the uncertainty on the political debate for new public expenses.

The functional to be minimized is then given by the expected value of the lagrangian:

$$(3) \qquad V_t(B_t, g_t) = \text{Min } E_t \{\sum_{j=0}^{+\infty} \delta^j \ \psi(\tau_{t+j}) + \lambda(S) [B_t - \sum_{j=0}^{+\infty} \delta^{j+1} (\tau_{t+j} - g_{t+j})]\}$$

which defines the value function conditioned on the evaluation period (t), given all the possible states of nature and the transversality condition for the debt as a consequence of the infinite horizon, $B_t \ \delta^t \rightarrow 0$ ($\rightarrow$ indicates the limit as t goes to infinity). In (3) all the terms considered, with the exception of the discount factor for what said above about r, are random variables included the Lagrange multiplier, $\lambda(S)$, which is depends on all the possible sequences (S) of the several states of nature that may occur through time.

The Bellman's equation associated to (3) (Sargent 1979, 1987) is given by

$$(3') \qquad V_t(.) = \text{Min}\, \psi(.) + \delta E_t V_{t+1}(.)$$

sub. (1)

and the first order conditions are

$$(4) \qquad \frac{d\psi(\tau_t)}{d\tau_t} = E_t \frac{d\psi(\tau_{t+1})}{d\tau_{t+1}} , \quad \forall\ t.$$

[3] Technically also g_t is a state variable but since it is exogenous –i.e., independent of the control variable- in that depends on the political debate, we name it as decisional.

From (4), considering the quadratic form of $\psi(.)$. We obtain

(4') $$\tau_t = E_t(\tau_{t+1})$$

and then

(5) $$\tau_{t+1} = \tau_t + e_{t+1}\ , \quad E_t(e_{t+1}) = 0$$

which means that the optimal taxation is distributed (~>) as a martingala process, which is stable on average but still $I(1)$. It is worth of notice that similar results might have been obtained also with other functional forms such as an isoelastic loss function or more generally by means of numerical calculus (Judd 1992, 1999).

By substituting (4') in the intertemporal budget constraint present in (3) and equating to zero, the control variable may be obtained as:

(6) $$\tau_t = r\ [B_t + \sum_{j=0}^{+\infty} \delta^{\,j+1}\, E_t(g_{t+j})]$$

or, iterating g_{t+j} from $t+j$ back to t,

(6') $$\tau_t = r\ [B_t + \sum_{j=0}^{+\infty} \delta^{\,j+1}\, E_t(\rho^j g_t + \sum_{s=0}^{j-1} \rho^j \varepsilon_{t+j-s}\ )]$$

and, developing the summation and applying the mean, we obtain

(7) $$\tau_t = rB_t + g_t\, r/(1+r-\rho).$$

Therefore, the average taxation rate - with respect to output (y_t)- implicit in the European SGP should be equal to

(8) $$\nu_t = r(B_t/y_t) + (g_t/\ y_t\ )r/(1+r-\rho).$$

We underline that from (8), in the absence of growth, the higher is the autoregressive coefficient the higher will be the optimal tax rate required by the SGP, which means that a more variable – or less under control- public expenditure should be monitored by a more painful level of taxation till reaching the rule of a balanced budget – i.e., taxation equal to total expenditure - in the case of $\rho = 1$.

3. *Consequences of violating the Stability and Growth Pact*

The consequences of not respecting the SGP for a country belonging to the monetary union are derived from (7) which depends on the initial conditions of the program and the exogenous parameters. Under the common view of financial crisis - due for instance to speculation or to a financial distress provoked by a rise in the interest rate or in the public expenditure - the solution for program (3) needs to be modified. How dangerous is it for the stability of the euro system? The answer depends upon the several factors pertaining the intensity and the frequency of such critical events. A part from the consideration that one of the reasons for the existence of the European Union is that it should be capable to face these events - which will then should be temporary-, the only remedy to such episodes is to run a new program under the new parameters and initial conditions that fit with the period of crisis. Nevertheless, we want to stress here that, even if it would be reasonable hoping to prevent or to ride out such crises, another kind of crisis, a systemic one, is anyway implicit in the simple hypotheses of the SGP underlying program (3). In fact, in order to obtain the dynamics of τ_t and the expressions e_t, it is possible to subtract from (6) the same expression for the taxation, τ_{t-1}, by remembering that, from (1), $B_t = B_{t-1} + g_{t-1} + B_{t-1}\, r - \tau_{t-1}$ and, from (7), $rB_{t-1}=\tau_{t-1} - g_{t-1}\, r/(1+r-\rho)$. In this way $B_t = B_{t-1} - g_{t-1}\, r/(1+r-\rho) + g_{t-1}$, so that

$$\tau_t - \tau_{t-1} = r\left[B_{t-1} + g_{t-1} - \frac{g_{t-1}r}{1+r-\rho} + \delta\sum_{j=0}^{+\infty}(\rho\delta)^j E_t g_{t+j}\right] - r\left[B_{t-1} + \delta\sum_{j=0}^{+\infty}(\rho\delta)^j E_{t-1} g_{t+j}\right]$$

from which

$$(9) \qquad \tau_t = \tau_{t-1} + \delta \sum_{j=0}^{+\infty} (\rho\delta)^j (g_t - \rho g_{t-1}) = \tau_{t-1} + \varepsilon_t\, r/(1+r-\rho).$$

This solution is viable only if the process, τ_t, is stable, that is true just in case of a new additional hypothesis on the limiting variance of ε_t and consisting in $\sigma^2_{\varepsilon,t} \to 0$. In the absence of this hypothesis the system will tend autonomously to crisis. Such a result strongly underlines the relevance of the decisional process on the public expenditure and especially the care that should be adopted to reduce the uncertainty on the expenditure decisions. This in practice calls for clear financial plans both in terms of public investments and services to provide. The crucial point is that the fiscal consequences of the unforeseen decisions, represented by the error terms on the right and side of (9), accumulate over time with the risk to overpass the maximum average taxation, ν^s, admissible by the society. This means that

$$(10) \qquad \tau_t - \nu^s y_t < 0,$$

which points out the level of the tax rate over which social conflicts and tensions would be so harsh to make the solution (6) and so the Euro system intolerable.

In the absence of significant output growth, this possibility becomes a certainty in our context given the martingala process (5) obtained for the solution of the control variable. In fact, for the accumulation of the error terms in (9), at some future point in time the probability that (10) occurs will be 1. Note that to reach such a conclusion just suffices the strict imposition of a stable AR(1) public expenditure and that an even worse result should occur under more tolerant (and maybe plausible) hypotheses. More specifically, in such cases v_t would surpass v^s in a shorter period of time. Therefore, considering the probability limit (*Plim*) the following proposition holds:

Proposition 1

Given the program (3), in the absence of growth, even if ε_t is i.i.d., $Plim[\tau_t / y_t - \nu^s \geq 0]=1$.

Also in this case of systemic crisis the only way to respect the stability pact is to run a new program under new values for the initial conditions of the decisional variable g_t, the state variable B_t the interest rate r and ρ. The new initial conditions may be obtained in several ways such as imposing a patrimonial tax or securization to reduce the debt, increasing the incentives to boost the production, y_t, and to reduce the taxation rate v_t, changing the conditions for the interest rate and rescheduling the dynamics of the public expenses. Without adventuring in the complexity of such choices, it is clear that they all are pure temporary devices as the Proposition 1 continues to be valid in that the time of crisis is just shifted forward. Actually, this is what is happening nowadays in Europe to those countries with more or less pronounced public debt problems: the political decision process of the annual financial budget should consist in decisions capable to control the fiscal pressure and not only in a mere revision of the initial conditions and the model parameters.

Therefore, changing initial conditions with a new *financial law* would only have the consequence of postponing the crisis.

4. *Relaxing the Stability and Growth Pact*

An attenuation of the SGP might be realized in two ways: a) by rising the upper bound of the 3% for the deficit/PIL ratio, b) by introducing exceptionally the financing of the deficit with money. The first possibility implies that $(B_t\text{-}B_{t\text{-}1})/y_t$ has more room to vary. However, since in program (3) it is not considered an upper bound for this term, the conclusions reached in the previous section are still valid. Furthermore, we just considered the hypothesis of stationarity for g_t as a measure adopted to control the deficit so that a more pronounced dynamics of this variable would worsen the dynamics of the taxation in the future and then would increase the possibility of occurring for condition (10). This is what actually has happened with the Greek crisis for the rising expenses and the impossibility of raising the taxation in that a higher level of fiscal pressure would be socially unsustainable.

Regarding the case b), it reflects the possibility of a financial support by the European Central Bank to the Governments that run into financial troubles –that is, the fiscal pressure is going to become socially unsustainable to pursue the requested policy actions- as it has recently occurred with the so called "quantitative easing" through which many billions of euros have been allocated in order to buy new Government bonds. Therefore, if such a possibility is allowed, the event of the systemic crisis is to be evaluated by the limit of the composed probability that the condition (10) occurs and that the injection of monetary base - with maximum amount per each period given by ΔM^{max} - provided by the ECB is not enough to cover the difference of the financial needs - i.e., the difference between taxes collected and the maximum fiscal revenue socially sustainable, $\nu^s y_t$. These two events are dependent and so the composed probability is given by

(11) $P[(\tau_t/y_t) \geq \nu^s]\ P[\ rB_t/\ y_t + (g_t\ /\ y_t\ )r/(1+r-\rho) - \nu^s \geq \Delta M^{max}/y_t\ |(\tau_t/y_t) \geq \nu^s]$, as $t \rightarrow +\infty$

The result of such a limit is clearly the same stated with the Proposition 1. In fact, Proposition 1 shows that the first probability of (11) tends to 1 and, for the same reasoning, this occurs also for the second probability since the accumulation of the unexpected public expenditure from the past will unavoidably go beyond the fixed amount ΔM^{max}. Therefore, this sort of financing is once again a temporary remedy even if the probability of a systemic crisis discussed in the proposition 1 is undoubtedly reduced by the second probability in (11). However, given that under our hypotheses the intervention of the ECB cannot occur if the taxation rate doesn't reach its threshold level – i.e., the two events are mutually exclusive- we may state that $P[\nu_t - \nu^s \geq \Delta M^{max}/y_t\ |(\tau_t/y_t) \geq \nu^s] = P[\nu_t \geq \nu^s + \Delta M^{max}/y_t\ |(\tau_t/y_t) \geq \nu^s]$ is equal to $P[\nu_t \geq \nu^s + \Delta M^{max}/y_t\ ]$ since $P[\{(\tau_t/y_t) \geq \nu^s\} \cap \{\Delta M^{max}/y_t = 0\} = \emptyset$. Then, it is possible to state the following proposition

<u>Proposition 2</u>

even if an attenuation of the stability pact, as defined in b), is introduced, it is true that

$$Plim[(\tau_t/y_t) \geq \nu^s]\ P[rB_t/\ y_t + (g_t\ /\ y_t\ )r/(1+r-\rho) - \nu^s \geq \Delta M^{max}/y_t\ |(\tau_t/y_t) \geq \nu^s] = P(\nu_t \geq \nu^s + \Delta M^{max}/y_t) = 1.$$

This last result allows drawing some first considerations on the rules imposed with the SGP. In particular, the "improvements" to the SGP are to be looked for not much in widening the strict parameters there established but rather in the way in which countries conduct and put into practice the political debate on public expenditure and taxes which, as the time passes, makes the sustainability conditions stated by (3) more and more compromised unless the necessary structural reforms in this line are pursued. Such reforms essentially should be of two general categories. 1) A clear and reliable expenditure plan to be carried on in the long run, whilst currently financial laws typically consider a modifiable expenditure plan with an horizon of only short term. The expenditure plan should be of long run and based on a broad social consensus on the main constitutional social rights and necessities. The broad social consensus is important in order to let the expenditure plan to be pursued independently of the Government in charge. Moreover, given that our model is particularly suitable for countries with high levels of fiscal pressure compared to the provided public services, the effectiveness of the public expenditure is a key variable for the determination of the above considered probability. In fact, it is well known that high fiscal revenue associated with a low quality of the public services provided are hardly compatible with sustained growth paths. Then, 2) a real improvement of the effectiveness, other than of the quantity, of the public services provided is advisable in order to monitoring the occurrence of (10).

5. *The dynamics of the state variable B*

The solution for the control variable gives the possibility to obtain the state of the system represented in (3) by the total public debt. From (7)

(12) $$B_t = [(\tau_t / r) - g_t /(1+r-\rho)]$$

which, given the autoregressive scheme for the public expenditure and the control variable, furnishes,

(13) $$E_{t_0}(B_t) = [(\tau_{t_0} / r) - \rho^t g_0/(1+r-\rho)]$$

that is,

$$\lim_{t \to +\infty} E_{t_0}(B_t) = (\tau_{t_0} / r) \tag{14}$$

In words (14) says that, given an initial time t_0, on average and in the long run the stock of debt must converge to a constant value. However, by iterating back in (12), it is possible to retrieve the stochastic expression for the stock of debt which is $I(1)$:

$$B_t = (\tau_{t_0} / r) - g_{t_0} [\rho^t / (1+r-\rho)] + \sum_{j=0}^{t-1} [1- \rho^j] \varepsilon_{t-j} 1/(1+r-\rho) \tag{15}$$

Such a result is extremely important since explains that, according to the SGP, the public debt tends to be stable on average but is still exposed to sudden changes in the expenses which - if not covered by taxation - engender a divergent process around a convergent deterministic path based on the initial conditions of public expenditure and taxation.
Therefore, neither the systemic failure of program (3) can be attributed to a public debt policy but, as said above, only to the unclearness of the policy expenditure program which makes unstable the behaviour of the control variable.
Note that, from (1), the public debt could seem $I(2)$. In fact,

$$B_t = (1+ r) B_{t-1} + g_{t-1} - \tau_{t-1}, \qquad 1+ r > 1 \tag{1'}$$

being τ_{t-1} ~> $I(1)$ and g_{t-1} ~> $I(0)$. However, we may write

$$B_t - B_{t-1} = r B_{t-1} + g_{t-1} - \tau_{t-1} \tag{1''}$$

and observe that, by virtue of expression (7), B_{t-1} and τ_{t-1} are cointegrated by the vector of coefficients $[r, -1]'$ with a cointegrating relation $g_{t-1}\, r/(1+r-\rho)$ which, combined with g_{t-1}, furnishes $g_{t-1} (1-\rho)/(1+r-\rho)$ which is still $I(0)$ and confirms that B_t is $I(1)$.

6. *Optimal refunding plan and debt dynamics by maturity*

In order to find the dynamics of the debt composition by maturity, program (3) needs to be further qualified. Specifically, it is required to consider the optimal reimbursement of debt according to maturity. This optimal technique implies that - once implemented by finding the appropriate taxation- the Government is in the condition to refund the debt in an optimal way upon request but doesn't imply that the reimbursement necessarily should happen. Given that the refunding occurs thanks to the control variable, the definition of the loss function of section 2 helps pursue this aim,

(16) $$Min\ L(.) = \psi({}_{t-1}B_t + {}_{t-2}B_t) + \delta\psi({}_{t-1}B_{t+1}),$$

where the term ${}_{t-1}B_t$ represents the short term debt ($B_{ST,\ t}$) issued at the period t-1 and expiring at period t, ${}_{t-2}B_t$ the long term one ($B_{LT,\ t}$) issued at period t-2 and expiring at period t and finally ${}_{t-1}B_{t+1}$ is the long term debt ($B_{LT,\ t+1}$) of period t-1 and expiring at t + 1. The discount factor is applied to the cost for the future loss of the tax collected for the reimbursement at time t + 1. The constraint, to be respected at any time t, is given by

(17) $$B_t = {}_{t-1}B_t + {}_{t-2}B_t + \delta\, {}_{t-1}B_{t+1}$$

and the first order conditions are

$$\psi'({}_{t-1}B_t + {}_{t-2}B_t) = \psi'({}_{t-1}B_{t+1})$$

or differently

(18) $${}_{t-1}B_t + {}_{t-2}B_t = {}_{t-1}B_{t+1}$$

which, by means of (17), gives

$${}_{t-1}B_{t+1} = (1 + \delta)^{-1} B_t \qquad \forall\ t$$

that, together with (7), furnishes the long term debt at time t+1 ($B_{LT,\ t+1}$)

(19) $$B_{LT,\, t+1} = [(\tau_t / r) - g_t /(1+r-\rho)]\,(1 + \delta)^{-1}$$

or, analogously to the previous section,

(19’) $$E_{t_0}(B_{LT,\, t+1}) = [(\tau_{t_0} / r) - \rho^t g_{t_0}][(1 + r)/(2 + r)].$$

Expression (19’) has an $I(1)$ representation

(20) $$B_{LT,\, t+1} = B_t\,(1 + \delta)^{-1} = [(\tau_0 / r) - \rho^t g_0 + \sum_{j=1}^{t-1} [\rho^j - 1 /(1+r-\rho)]\, \varepsilon_{t-j}$$

and

(21) $$\lim_{t \to +\infty} E_{t_0}(B_{LT,\, t}) = (\tau_{t_0} / r)[(1 + r)/(2 + r)] \quad \forall\, t$$

from which it is possible to derive the rule that the long term debt (B_{LT}) in the long run - i.e., in the limit- and on average must reach a constant value. Differently, the short term debt ($B_{ST,\, t}$) is stationary and must tend to 0 on average as time passes. In fact, by first differencing $B_{LT,\, t}$ in (19) it is possible to obtain $B_{ST,\, t}$ after substituting in (18)

(19’’) $$B_{ST,\, t} = [(e_t / r) – (g_t – g_{t-1})/(1+r-\rho)]\,(1 + \delta)^{-1}$$

which is proved to be $I(0)$. Furthermore, given that (21) holds for each t and using (18), it is possible to obtain

(22) $$\lim_{t \to +\infty} E_0(B_{ST,\, t}) = 0$$

which in practice means that the short term debt should be kept at a level only temporarily different from 0 as should naturally be, and in fact the original function of the short term debt was that of settling the occasional cash flows of the Government budget.

7. *Empirical evaluation of the SGP failure risk probability*

The empirical analysis we are interested in is on the validity of the Proposition 2, which may be tested by implementing a probability model for the estimation of Prob($v_t \geq v^s + \Delta M^{max}/y_t$). More specifically, we want to evaluate the probability that the control variable v_t goes beyond the threshold level given by the socially admissible taxation rate v^s plus – possibly - the financial aid $\Delta M^{max}/y_t$, which would mean the failure to comply the SGP. Of course we don't know exactly how much is such a threshold, nonetheless we may reasonably argue that "high" rates of v_t may include both the mentioned terms. Therefore, since there isn't an empirical evidence of the countries' SGP failure neither the values of the control variable v_t are a priori known, we implement two possible probability models by assigning for the occurrence of the failure value 1 when the control is *supposed* to be reasonably equal or greater than $v^s + \Delta M^{max}/y_t$ and 0 when is lower. We consider two cases of reference when the former is supposed to occur. Specifically, when the total Government expenditure upon output, $G_t/y_t = r\ B_t/y_t + g_t/y_t$, is equal or greater then either 52% or 55%. We arrive to such a choice experimentally by running preliminarily several panel consistent estimations of equation (2) which all confirm a coefficient ρ below 1 though comprised between 0.8 and 0.9. Therefore, recalling from (8) that *in the worst case* of ρ =1 also the coefficient of g_t, $r/(1+r-\rho)$, is equal to 1 and the control variable equates the total public expenditure, we deem that the thresholds above indicated may represent two experimental values for the total public expenditure which are large enough to pinpoint the cases when $v^s + \Delta M^{max}/y_t$ is reached or over passed by the *unknown* value of v_t. For this reason we fixed the second threshold particularly high in order to be prudent in case that the value of ρ, descending from the theory of problem (3) combined with (10) - expressed by proposition 2-, would be even lower than that one obtained by the preliminary regressions. [4] Therefore, also the results of the empirical analysis will be prudent in this sense.

We tried both a Logit and a Probit model obtaining very similar results. However, to be prudent again, we present in Table 1 the former empirical model in that the corresponding probability outcome of SGP failure is slightly lower than that one of the latter.[5] We estimate a panel data with random effect model validated by the Chi2 test (likelihood ratio test) shown in Table 1 and coherently with the formulation of equations (8) and (9) which

[4] Also in this case, in order not overload the presentation of the econometric analysis, this empirical result is available upon request.

are characterized by the error term. We consider 12 European countries - Italy, Belgium, Germany, Greece, Spain, France, Ireland, Luxemburg, Netherlands, Austria, Portugal, Finland - and 30 years ranging from 1984 till 2013. Interestingly, the constant term represents the *a priori* unknown threshold level of the control variable -included the monetary financing if occurred- associated to the threshold level of the "latent" variable – i.e., the total public expenditure.[6] In fact, our probability model is

(23) $P[v_t \geq v^s + \Delta M^{max}/y_t | \, \mathbf{x}_{t-1}] = P[-(v^s + \Delta M^{max}/y_t) + r\ B_t/y_t + (r/(1+r-\rho))\ g_t/y_t \geq 0 \mid \mathbf{x}_{t-1}]$, with $\mathbf{x}_t' = [B_t/y_t,\ g_t/y_t]$,

to which is associated, for the i-th country, the following likelihood to be maximized,

(24) $L_i = \Pr \mathrm{ob}\left(fail_{i1},....., fail_{iT} \,|\mathbf{X}\right) = \int_{\inf_{i1}}^{\sup_{i1}} ..... \int_{\inf_{iT}}^{\sup_{iT}} f\left(\varepsilon_{i1},..., \varepsilon_{iT}\right) d\varepsilon_{i1},...d\varepsilon_{iT}$, i=1,…, N; t=1,…T

where **X** is the dataset of the covariates for every time and country and the possibility of failure to comply SGP – i.e., the dependent variable - is represented by $fail_{it}$ =1 or otherwise 0. In the former case, by defining $\boldsymbol{\beta}' = [r,\ r/(1+r-\rho]$, $(\inf_{it}, \sup_{it}) = (-\infty, \mathbf{x}_{it}'\boldsymbol{\beta})$, in the latter $(-\mathbf{x}_{it}'\boldsymbol{\beta}, +\infty)$.

The random effect model implies that the residual of the theoretical model is $\varepsilon_{it} = \eta_{it} + u_i$, where, as usual, the random variables on the right hand side are independent and with the following proprieties

$E[\eta_{it}|\mathbf{X}]=0$, $Cov[\eta_{it}, \eta_{js}|\mathbf{X}]=Var[\eta_{it}|\mathbf{X}]=1$ if $i = j$ and $t = s$, 0 otherwise
$E[u_i|\mathbf{X}]=0$, $Cov[u_i, u_j|\mathbf{X}]=Var[u_i|\mathbf{X}]=\sigma^2_u$ if $i = j$, 0 otherwise
$Cov[\eta_{it}, u_j|\mathbf{X}]=0\ \forall\ i, t, j$, $E[\varepsilon_{it}|\mathbf{X}]=0$, $=Var[\varepsilon_{it}\,|\mathbf{X}]=1+\sigma^2_u$, $Cov[\varepsilon_{it}, \varepsilon_{js}\,|\mathbf{X}]= \sigma^2_u$.

Expression (24) is first simplified and then maximized by means of the Gauss-Hermite quadrature. The simplification adopted it is based on the joint distribution there represented, which may be considered in terms of the integral over u_i of the product of the single independent densities conditioned on u_i -which is the term engendering the dependence over time for the i-th observation. This, after some manipulations, brings to

[5] Results of the Probit model are eventually available upon request.
[6] As usual, for forecasting purposes such a value, being a point estimate, is to be incremented with the double of its standard error.

$$(25)\ \ L_i = \Pr\mathrm{ob}\left(fail_{i1},\ldots\ldots, fail_{iT} \left| \mathbf{X} \right.\right) = \int_{-\infty}^{+\infty} \prod_{t=1}^{T} \left(\int_{\inf_{it}}^{\sup_{it}} f\left(\varepsilon_{it} \left| u_i \right.\right) d\varepsilon_{it} \right) f\left(u_i\right) du_i\ .$$

Finally, we used the Butler and Moffitt's method, which consists in assuming a normal distribution for u_i, and obtain, assigning to $f(.)$ the logistic function,

$$(26)\ \ L_i = \Pr\mathrm{ob}\left(fail_{i1},\ldots\ldots, fail_{iT} \left| \mathbf{X} \right.\right) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-\left(\frac{u_i}{\sigma_u \sqrt{2}}\right)^2} \left[\prod_{t=1}^{T} \frac{e^{\left(2\, fail_{it} - 1\right)\left(\mathbf{x}_t^{'}\boldsymbol{\beta} + u_i\right)}}{1 + e^{\left(2\, fail_{it} - 1\right)\left(\mathbf{x}_t^{'}\boldsymbol{\beta} + u_i\right)_i}} \right] d\left(\frac{u_i}{\sigma_u \sqrt{2}} \right),$$

which is maximized with the above mentioned numerical procedure.[7]

Table 1. Estimation of Logit model. Point estimates, RE effects.

regressors	*Parameters*	$G_t/y_t \geq 52\%$: $v_t \geq v^s$	$G_t/y_t \geq 55\%$: $v_t \geq v^s$	$G_t/y_t \geq 52\%$: $v_t \geq v^s$	$G_t/y_t \geq 55\%$: $v_t \geq v^s$
g/y	$r/(1+r-\rho)$	0.777***	0.614***	1	1
	S.E.	0.109	0.096	-	-
B/y	r	0.058***	0.039 ***	0.071***	0.058***
	S.E.	0.013	0.011	0.014	0.013
$v^s + \Delta M^{max}/y_t$	α_0	-41.449***	-34.680***	-52.775***	-55.402***
	S.E.	5.709	5.228	1.274	1.323
-	ρ	0.983	0.944	1	1
Chi2 test of μ=0 (P-value)		0.004***	0.044**	0.000***	0.006***
Log likelihood		-80.047	-59.467	-81.677	-64.664
R^2 (likelihood ratio index)		0.51	0.49	0.50	0.44

***Significance at 99%, **significance at 95%. μ is the proportion of the total variance contributed by the panel-level variance component.

From the estimated coefficients we observe that they unequivocally decrease once the failure condition is more restrictive. This is quite reasonable in that the probability of failure itself decreases and therefore, in order for this to be verified, also the estimated

[7] We implement this calculus using Stata 12, the remaining part of the statistical analysis has been developed with Gretl.

interest rate and the coefficient of the real expenditure decrease. For analogous reasons, we obtain greater estimated interest rates once we constrain ρ to 1. Notably, the upper limit of the tax rate for the society – eventually augmented with the monetary financing from the ECB – is about 50% by considering the constant term plus the double of the standard error[8], whilst the interest rate on the public debt ranges from the 4% till about 7%. The autoregressive component, ρ, is quite high even if not constrained to 1.

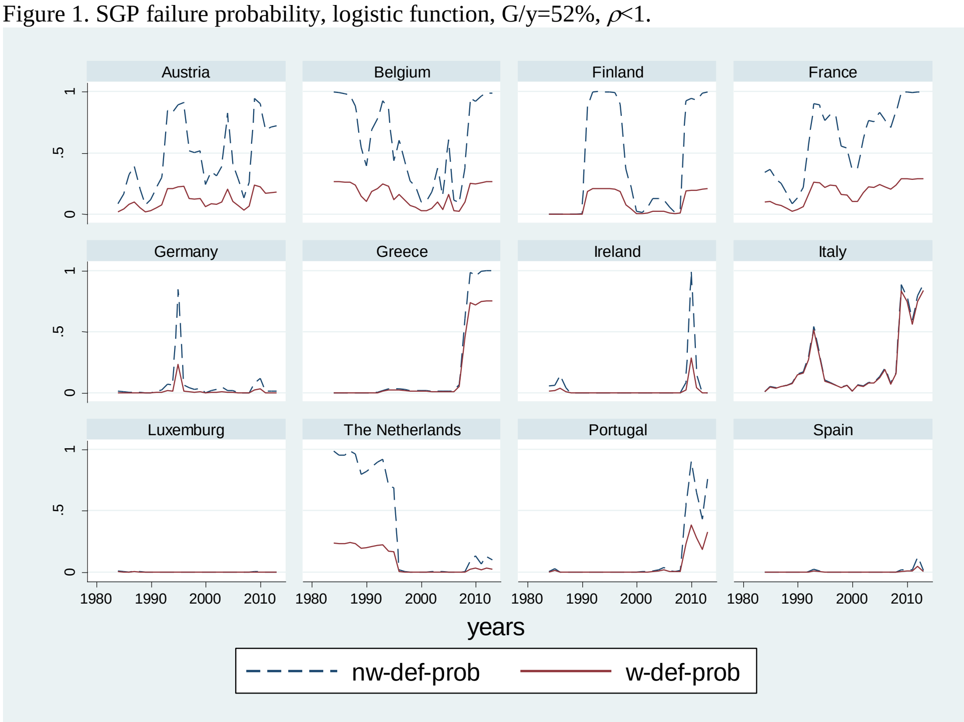
As above mentioned, in developing such argumentations, and in particular to make valid comparisons among countries, we must take into account the effectiveness of the public expenditure perceived by people because, as said above, the probability to comply with SGP is to be evaluated in relationship with the credibility that the Governments have to provide public services with a level of quality in accordance with the commitments undertaken. We consider such an aspect by comparing the failure probability obtained by the estimated models of Table 1 with the one corrected with the Worldwide Governance Indicator (WGI 2014), which theoretically belongs to $[0, +\infty)$ and reflects the Government effectiveness in terms of policy formulation and implementation - and so concerns the perceptions of the quality of public services and Government's credibility. We divide the WGI index per each country by that one of the country at the minimum level and weight the failure probability accordingly to the reasoning that for the country in the worst condition in implementing the expenditure policy there is no possibility of attenuating the SGP failure probability. Therefore, we have estimated the probability to surpass a common fiscal upper threshold level - which is, for what said above, a quite high percentage of output – and then on the base of the degree of effectiveness in providing services mitigate the probability of failure.

In the following figures 1-4 the dashed line represents the non weighted failure probability while the continuous line the weighted one. It is worthwhile observing that for many countries there is a substantial difference between these two consisting in a much lower

[8] Moreover, v^s should be corrected – in case of existence- by the yearly constant public expenditure normalized by a discount factor. This means that the socially admissible tax rate would be higher than what reported in Table 1 as shown in the Appendix.

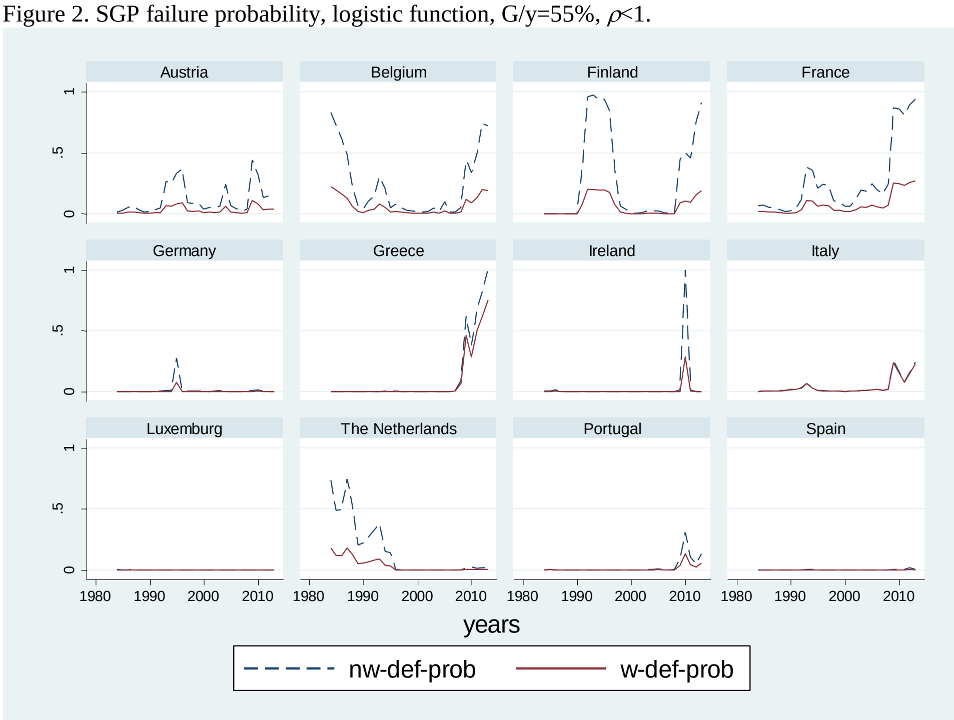
weighted failure probability. In particular, this occurs for some countries at a first sight in critical conditions like France, Austria and Finland with a high non weighted failure probability. For such countries we may assert that the failure probability is even lower than that one of other countries like Italy which has a lower non weighted probability. The country in the most critical condition is Greece since 2009. For such a country, even if the effectiveness of the Government policies improves from that period, the failure probability still keeps on being high. As for Italy, in practice there is no difference between the two lines of the following figures thus showing that the high taxation, consequent to the high public expenditure, is not perceived as the duly payment for the services provided by the Government. For this country the failure probability decreases very much with the threshold level of 55%. The hope that this is the correct threshold of reference might be suggested by the level of the log likelihood of Table 1 which is higher for such a value for both the cases of ρ constrained to 1 or not.

Figure 1. SGP failure probability, logistic function, G/y=52%, ρ<1.



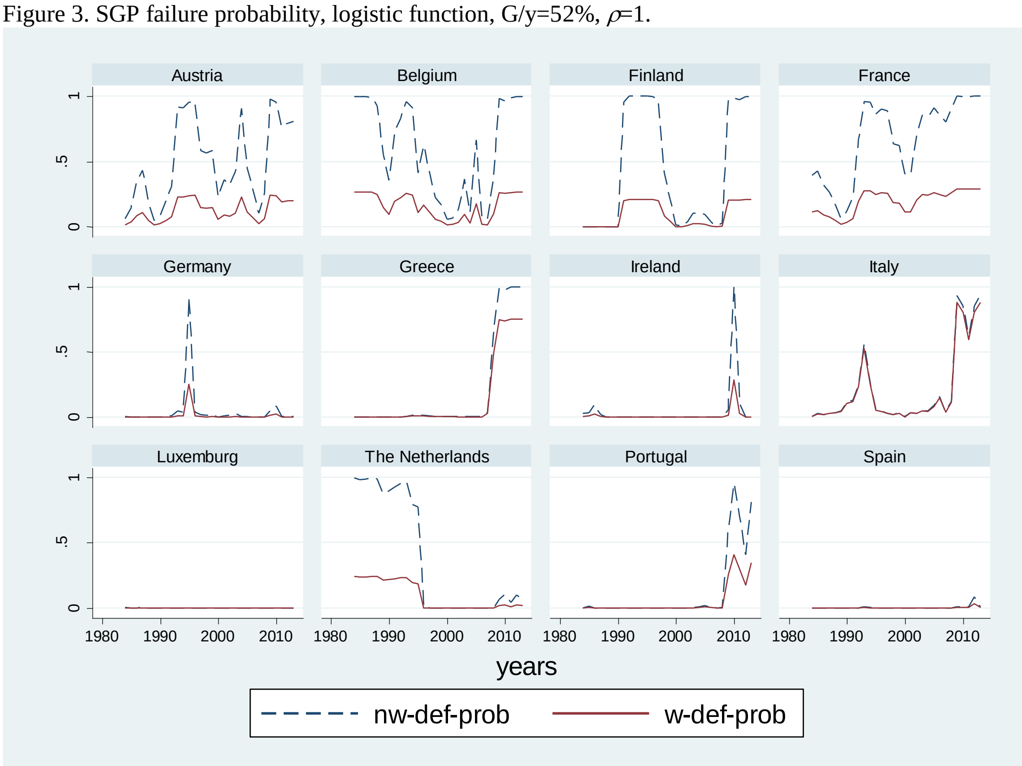
Non weighted (nw-def-prob) versus weighted (w-def-prob) failure probability.

Figure 2. SGP failure probability, logistic function, G/y=55%, ρ<1.



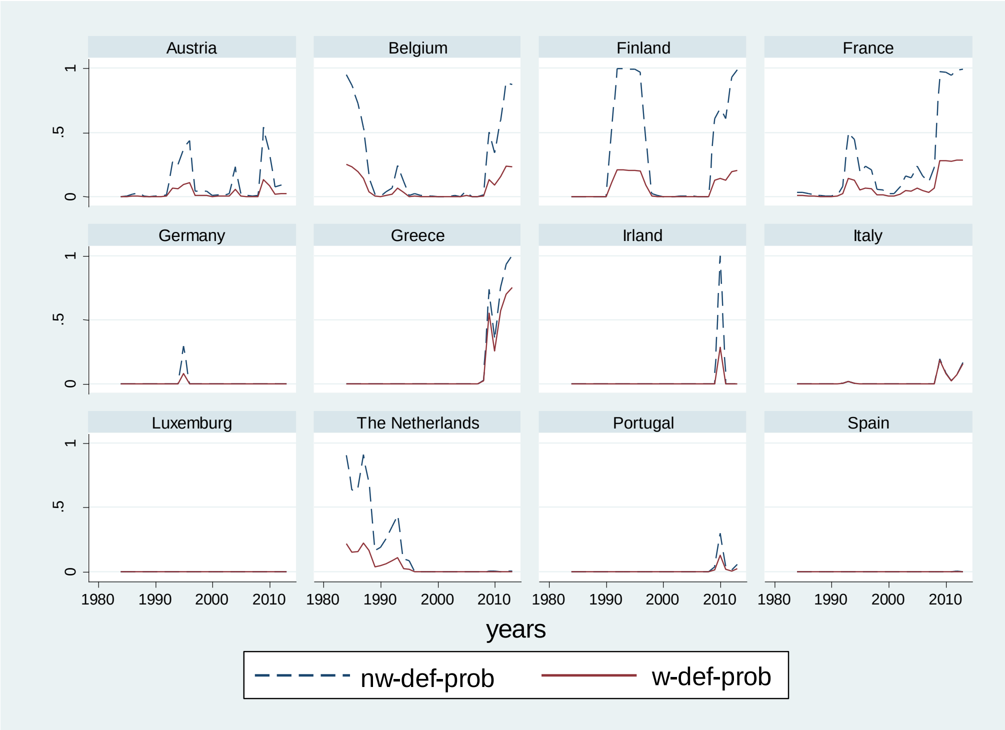
Non weighted (nw-def-prob) versus weighted (w-def-prob) failure probability.

Figure 3. SGP failure probability, logistic function, G/y=52%, ρ=1.



Non weighted (nw-def-prob) versus weighted (w-def-prob) failure probability

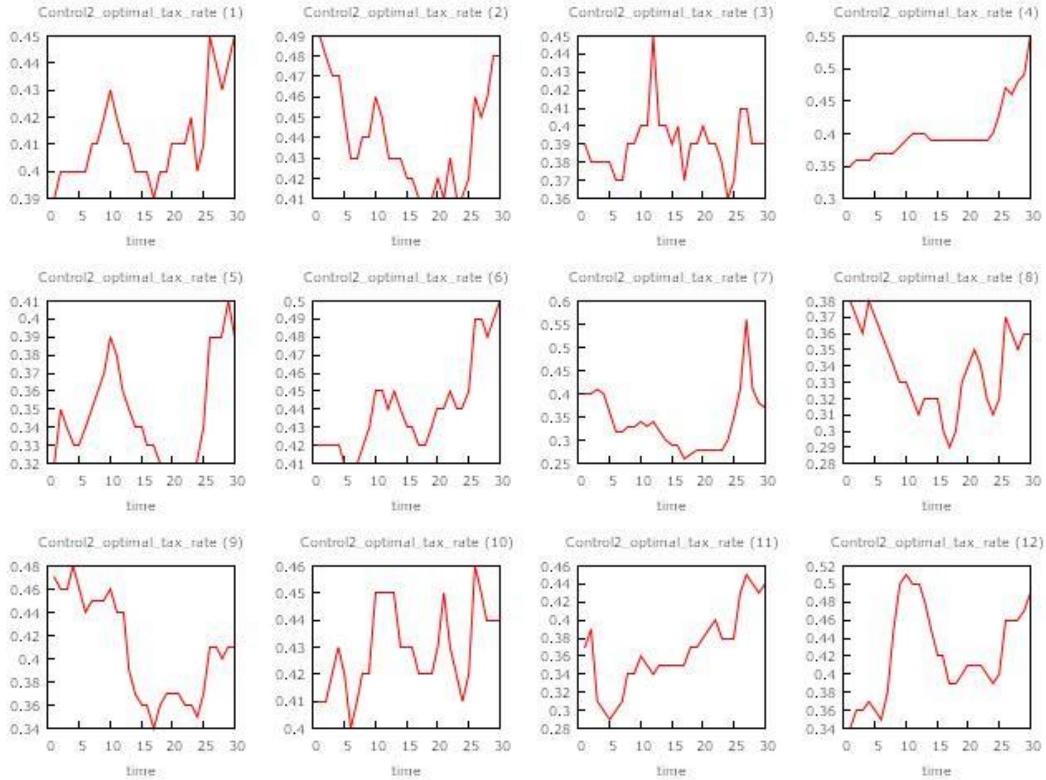
Figure 4. SGP failure probability, logistic function, G/y=55%, ρ=1.

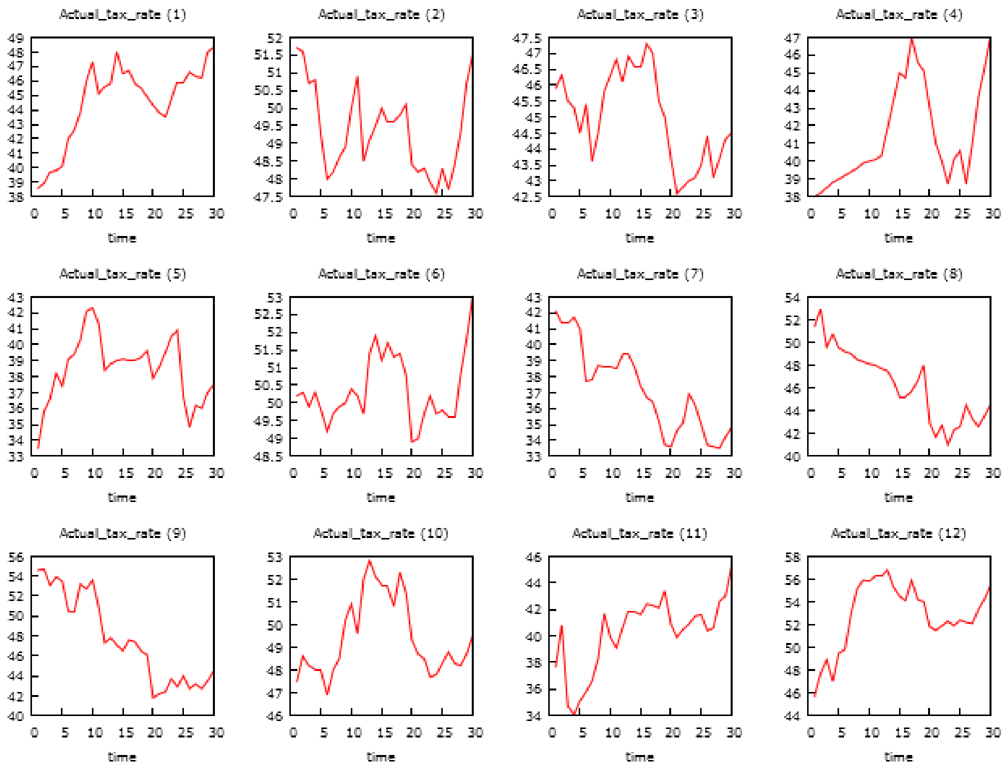


Non weighted (nw-def-prob) versus weighted (w-def-prob) failure probability.

Here below in the first half of Figure 5 we show the path of the control variable under the scenario G/y=55% which has been found less painful both for the level of the interest rate and for the autoregressive coefficient of the real public expenditure. Moreover, as recalled above, such a scenario reflects the highest probability to occur. As one may easily check, almost all countries with the exception of Germany – and less evidently of The Netherlands and Ireland - have increasing trends of the control requested to respect the Stability and Growth Pact, which is, together with the probabilities calculated in the figures 1-4, an alarming indications of the Euro system vulnerability. Of course, a relevant hypothesis at the basis of these results is that output is in difficulty of growing. Nonetheless it means that growth is an imperative condition for the sustainability of the Euro system as conceived in the SGP but at the same time may not be given for grant! The other measures to survive are to change the initial conditions and parameters of program (3), unless countries reach the stability in terms of decisions on the real public expenditure. A stable public expenditure decisional process would produce a stable control variable in terms of tax rate also during periods of low growth paths, thus allowing the feasibility of program (3). It is straightforward noticing that this implies transparent plans of expenses based on a public programming that should be shared on a large scale consensus capable to overpass the alternation of the politic parties at least as far as the main categories of g_t are concerned. Such a transparent policy of expenses should concern g_t as a whole – i.e., both from the current and the investment side. The second half of Figure 5 confirms the above mentioned criticalities in that even if countries like Spain or Portugal show paths of control variables capable to stay under the critical threshold, the actual values of the tax rates are well below the control variable itself. This is a clear indication that two are the requisites necessary to implement the SGP: from one side the taxes implicitly prescribed by the SGP should be socially sustainable in comparisons with the public services provided by the single countries, from the other side countries should be able to respect such prescribed taxes.

Figure 5. Optimal control and actual tax rate





(1)= Italy, (2)= Belgium, (3)= Germany, (4)= Greece, (5)= Spain, (6)= France, (7)= Ireland, (8)= Luxemburg, (9)= Netherlands, (10)= Austria, (11)= Portugal, (12)= Finland; time 1 = 1984, time 30 = 2013; Control 2 = average tax rate when the latent variable (G/y) threshold is 55% and $\rho < 1$.

We conclude this empirical section by remarking that even in the case we choose prudentially a probability model without the constraint of $\rho = 1$ - which lowers the control of the taxation *below* the level of the total public expenditure – the feasibility of the optimal taxation deriving from the SGP cannot be able to leave output growth out of consideration. In fact, Figure 5 shows an alarming dynamics of v_t for the absence of a sustained growth for all countries with the only exception of Germany, which is notably the leading European country at the present. Therefore, what is strongly required by countries in order to be protected by the possibility of a recession, which may potentially engender the failure of the SGP, is the clearness of an expenditure plan of long run so as to reduce the uncertainty of undesired sudden expenses which would influence negatively the path of the taxation rate. Moreover, from Table 1, we observe the necessity to reduce the autoregressive component of the public expenditure which is next to the nonstationary case. Reducing such a component will reduce the variability of the taxation and so will increase the probability to survive of the SGP in the absence of growth.

8. *Conclusions and further remarks*

In this research an optimal control model has been presented to show how weak is the European system for those countries that have problems with a heavy level of taxation and low growth in absence of appropriate services. This provokes social tensions that make effective the constraint of the admissible social taxation rate and, as time passes, put in critical condition the maintenance of the Euro system for those countries which incur in such criticalities. From the empirical side only few European countries show a stable tax rate trend requested by the SGP. The main reason is linked to the public expenditure that reflects the countries' political process. If such a process is uncertain for the future, such uncertainty will be cumulated over time by the taxes engendering the serious risk of not meeting the budget constraint of the program coherent with the respect of the monetary union. As this process is autonomous, unless of changes in the initial conditions of the program, it will force countries to abandon the system even in the privilege of less strict rules of the stability pact. Therefore, the structural reforms requested for the feasibility of the Union are to be searched in connection with the sources of uncertainty of the public expenditure. These basically may be summarized in unjustified measures of expenditures cuts that later are to be paid or by the expenses hidden to the community when institutions profit of their autonomy or when such expenses are indirectly linked to the public sector as in the case of the privatized public companies in financial trouble that Governments are forced to help in that have strong influence on the public utility. This means that whilst in the pre-euro era such a way of managing the public expenses could be financed in several manners such as by large deficits, money and devaluation and, hopefully, output growth, with the Monetary Union there remains only output growth. The consequence of such a result is the conclusion that without output growth the chances for the European system to survive could be seriously compromised unless the above mentioned uncertainty in the political process is reduced. Finally, a derivation of the composition and the time stochastic behaviour of the public debt according to maturity are obtained from the consideration of an optimal reimbursement plan.

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References

Alesina A., Prati A., Tabellini G. (1990), Public Confidence and Debt Management: a Model and a Case Study of Italy, in Dornbush R., Draghi M. (a cura di), *Public Debt Management: Theory and History*, Cambridge, Cambridge University Press, pp. 94-124.

Barro, R., J., (1979), On the Determination of the Public Debt, *Journal of Political Economy*, vol. 87, n°. 5, pp. 940-971.

Judd, K.L. (1992) Optimal Taxation in Dynamic Stochastic Economies: Theory and Evidence. Mimeo, Hoover Institution.

Judd, K., (1999), Optimal taxation and spending in general competitive growth models, *Journal of Public Economics*, n° 71, pp. 1-26.

Obstfeld M. (1986), Rational and Self-fulfilling Balance of Payments Crisis, *American Economic Review*, n. 76, pp. 72-81.

Obstfeld M. (1994), The Logic of Currency Crises, in *Cahiers Économiques et Monétaires* (Banque de France), n. 43, pp. 189-213.

Obstfeld, M., (2013), On Keeping Your Powder Dry: Fiscal Foundations of Financial and Price Stability, Discussion Paper No. 2013-E-8, *IMES*, Bank of Japan.

Galina, H., Obstfeld, M., (2014), The Euro and the geography of international debt flows, *NBER* Working Paper 20033.

Sargent T. J. (1979), *Macroeconomic Theory*, Academic Press.

Sargent T. J. (1987), *Dynamic Macroeconomic Theory*, Harvard University Press.

Tanner, E., Carey, K., (2005), The Perils of Tax Smoothing: Sustainable Fiscal Policy with Random Shocks to Permanent Output, *IMF Working Paper*, WP/05/207.

WGI, (2014), www.govindicators.org

Appendix

In this Appendix we study the solution of problem (3) when

A.1 $g_t = g_c + \rho\, g_{t-1} + \varepsilon_t,\ |\rho|<1,\ E_t(\varepsilon_{t+1}) = 0,\ \ \varepsilon_t \sim \text{i.i.d},\ g_{t_0} \geq 0,\ g_c > 0.$

where g_c is a constant value which accounts for the fixed expenses that may be present every year. In this case

A.2 $$\tau_t = r\,[B_t + \sum_{j=0}^{+\infty} \delta^{\,j+1} E_t(\rho^j g_t + g_c\, \frac{1-\rho^j}{1-\rho} + \sum_{s=0}^{j-1} \rho^j \varepsilon_{t+j-s}\,)]$$

from which

A.3 $$\tau_t = rB_t + (g_t\, r + g_c)/(1+r-\rho)$$

A.4 $$\nu_t = r(B_t/y_t) + (rg_t + g_c)\,/\,y_t\,(1+r-\rho).$$

However, the dynamics of τ_t remains unaffected by g_c. In fact,

A.5 $$\tau_t - \tau_{t-1} = r\,(B_t - B_{t-1}) + (g_t + g_{t-1})r/(1+r-\rho)$$

but

A.6 $$B_t = B_{t-1} + [\,\tau_{t-1} - g_{t-1}r/(1+r-\rho) - g_c/(1+r-\rho)] + g_{t-1} - \tau_{t-1}$$

where $r\,B_{t-1} = [\,\tau_{t-1} - g_{t-1}r/(1+r-\rho) - g_c/(1+r-\rho)]$, so that

A.7 $\tau_t - \tau_{t-1} = (g_t - \rho\, g_{t-1})(1-\rho)r/(1+r-\rho) - g_c\, r/(1+r-\rho)$ with $g_t - \rho\, g_{t-1} = \varepsilon_t + g_c\, r/(1+r-\rho)$

from which is obtained for τ_t the same dynamics as (9).

As for the public debt, from A.6 its dynamics is given by

A.8 $$B_t = B_{t-1} + g_{t-1}(1-\rho)/(1+r-\rho) - g_c/(1+r-\rho)$$

from which, iterating back the public expenditure, we obtain

A.9 $$B_t = B_{t-1} + g_{t0}\,\rho^{\,t-1}(1-\rho)/(1+r-\rho) - \rho^{\,t-1} g_c/(1+r-\rho) + \frac{1-\rho}{1+r-\rho}\sum_{j=0}^{t-2} \rho^j \varepsilon_{t-j+1}$$

and, iterating back the public debt

A.10 $B_t = \ B_0 + \sum_{j=0}^{t-1} \rho^j g_{t_0} \frac{1-\rho}{1+r-\rho} - \sum_{j=0}^{t-1} \rho^j \frac{g_c}{1+r-\rho} + \frac{1-\rho}{1+r-\rho} \sum_{s=1}^{t} \sum_{j=0}^{t-s} \rho^j \varepsilon_{t-s-j}$

but, given that $\frac{1-\rho}{1+r-\rho} \sum_{s=1}^{t} \sum_{j=0}^{t-s} \rho^j \varepsilon_{t-s-j} = \sum_{j=1}^{t-1} \left(1-\rho^j\right) \varepsilon_{t-j}$, and that, by virtue of A.3, $B_0 = \frac{\tau_0}{r} - \frac{g_{t_0}}{1+r-\rho} - \frac{g_c}{r\left(1+r-\rho\right)}$ we obtain

A.11 $B_t = \ \frac{\tau_0}{r} + \rho^t \frac{g_c - g_{t_0}\left(1-\rho\right)}{\left(1-\rho\right)\left(1+r-\rho\right)} - \frac{g_c}{r\left(1-\rho\right)} + \sum_{j=1}^{t-1} \left(1-\rho^j\right) \varepsilon_{t-j}$.

As observed in the text, in case of existence of g_c one should add the term $(g_c/y_t)/(1+r-\rho)$ to the constant value estimated in Table 1 in order to obtain v^s –eventually plus $\Delta M^{max}/y_t$. In fact, in such a case, we would have estimated the model

A.12 $P[r\, B_t/y_t + (r/(1+r-\rho))\, g_t/y_t \geq v^s + \Delta M^{max}/y_t - (g_c/\, y_t)/(1+r-\rho) \mid \mathbf{x}_{t-1}]$

where the constant excludes the term under question. Such a term is unknown but ranges from about 4% till 9% from the preliminary estimates we conducted on g_t according to the method used.