



SAPIENZA
UNIVERSITÀ DI ROMA

Dipartimento di Scienze Statistiche
Sezione di Statistica Economica ed Econometria

Elisa Fusco Bernardo Maggi

Bank financial world crisis: inefficiencies and responsibilities

*DSS Empirical Economics and Econometrics
Working Papers Series*

DSS-E3 WP 2016/2

Dipartimento di Scienze Statistiche
Sezione di Statistica Economica ed Econometria
“Sapienza” Università di Roma
P.le A. Moro 5 – 00185 Roma - Italia

<http://www.dss.uniroma1.it>

Bank Financial world crisis: Inefficiencies and Responsibilities

Elisa Fusco^a, Bernardo Maggi^{b,*}

^a*University of Rome La Sapienza*

^b*Department of Statistical Sciences, University of Rome La Sapienza, Faculty of
Engineering, Informatics and Statistics*

Abstract

In light of the recent financial world crisis, is crucial to investigate into the responsibilities of the main actors in the credit sectors, *i.e.* banks and local governments.

In this framework, we propose a methodology able to analyze the quality of the problem loans adopted by banks, their level of efficiency in the risk management strategies and the governments policy action in the supervision of the local banking system. Our approach is based on the introduction of the “Non performing Loans” variable as an undesirable output in an output distance function (as stochastic frontier) in order to estimate the efficiency of the bank and calculate the shadow price of the *NPLs* (not normally observable) per each year, bank and country. Then we compare the management of the *NPLs* and their price across geographic areas and bank dimension over time in order to map the responsibilities and to draw some policy implications.

From an econometric point of view, we -to our knowledge- for first adopt the semi-nonparametric Fourier specification which, among the functional-flexible-form alternatives, is capable to guarantee the convergence of the estimated parameters and the related X-efficiency to the true ones.

Keywords: Commercial bank, Financial world crisis, Non performing loans, Efficiency, Flexible forms, Distance function

JEL classification: *G21, D24, C33, C51, L23.*

*Corresponding author.

Email addresses: elisa.fusco@uniroma1.it (Elisa Fusco),
bernardo.maggi@uniroma1.it (Bernardo Maggi)

1. Introduction

Till recently, most part of the literature of banking systems efficiency neglected the question of problem loans. Under the influence of the 2008-9 crisis, such a question started having growing consideration. [Berger and DeYoung \(1997\)](#) pioneered this field trying to face the study of the relations between problem loans and efficiency by means of the Granger-causality method, [Hughes and Mester \(1993\)](#) considered problem loans inside the frontier function. However, both of the attempts are not satisfactory because, the former is a mere statistical tool based on the VAR methodology and so deprived from an economic interpretation of causality, the latter is incoherent since an increase of efficiency may be due simply by increasing number of regressors. Only at the beginning of the first decade of 20s with the works of [Pastor \(2002\)](#) and [Pastor and Serrano \(2005\)](#) the question under consideration has been addressed properly with a non parametric approach, which is of less powerful insight from the modelization point of view. [Pastor and Serrano \(2006\)](#) adopted a parametric approach but did not find a functional relationship between stochastic frontier and non performing loans (*NPLs*) and focused their investigation on the connection between *NPLs* and X-efficiency. [Maggi and Guida \(2011\)](#) addressed this point by considering an indirect function linking *NPLs* with stochastic frontier.

With the present work we go further by inserting directly the *NPLs* variable in the stochastic frontier as a negative output, taking advantage of the fact that our definition of efficiency relies on the concept of the distance function. In such a way we are capable to asses on the quality of the problem loans adopted by banks and on their responsibility in the risk management. The former question is addressed by calculating the price of non performing loans per each year, bank and country considered in our dataset, the latter by comparing the management - in terms of variance analysis - of *NPLs* and their price across geographic areas and bank dimension over time. In doing so we provide a methodology which allows from one hand to alert in advance on an incumbent state of crisis and, from the other hand to evaluate the responsibility to be imputed to the main actors in the credit sectors, *i.e.* banks and local governments. Furthermore, an economic policy in terms of regulatory activities focused on the *NPLs* price comes out naturally as an implication of the analysis implemented. In fact, notably, the *NPLs* price is unknown and therefore not normally observable. Instead, our methodology allows to calculate it from the first order conditions underlying the estimation performed. Indeed, also in [Maggi and Guida \(2011\)](#) there is the possibility to evaluate a similar indicator. However, from the cost function there considered, the marginal cost calculated cannot be assumed as a price-quality

indicator in that this would have been possible only with perfect competition which is not the case for credit market. Moreover, that methodology passes through the definition of a density function which inevitably involves a degree of arbitrariness in its form of definition. From an econometric point of view, we -to our knowledge- for first adopt the semi-nonparametric Fourier specification which, among the functional-flexible-form alternatives, is capable to guarantee the convergence of the estimated parameters and the related X-efficiency to the true ones ([Gallant \(1981\)](#), [Berger et al. \(1997\)](#)).

Then our goals consist in: 1) finding a map of the responsibilities of the last financial crisis, 2) finding a road to regulate the risk in the credit market, 3) alerting the crisis period, 4) providing a rigorous method to calculate efficiency in case of a production function with undesired outputs. In order to cope with exigency of monitoring the credit sector, for what said above, our prime necessity is to calculate the shadow-price applied to *NPLs*. Hence, in the second section we define the theoretical framework of the model, where the optimizing behavior of the banks in charging prices is described. In the third section we describe our dataset and variables. In the fourth section, we derive, specifically for the non performing loans the analytical expression of such a price and the analytical form used in the empirical analysis. In the fifth section, we estimate the distance-revenue function. In the sixth section, we obtain the evaluation of the X-efficiency and the distribution of the *NPLs* price, which let us detect the outcoming responsibilities and omissions in the monitoring processes relatively to the last financial world crisis. The seventh section draws some comments on the main results with the policy implications. The eight section concludes.

2. Theoretical model

In this section we present the model for the determination of banks outputs prices. We intend to derive the X-inefficiency and a closed form solution for price of problem loans conceived as a negative output. Such a closed form will be used for the estimation in the next section. The representative commercial bank uses a positive vector of N inputs, denoted by $\mathbf{x} = (x_1, \dots, x_N)$, $\mathbf{x} \in \mathbb{R}_+^N$ to produce a positive vector of M outputs, denoted by $\mathbf{u} = (u_1, \dots, u_M)$, $\mathbf{u} \in \mathbb{R}_+^M$. The production technology of the bank can be defined by the output set, $P(\mathbf{x})$ that can be produced by means of the input vector \mathbf{x} , *i.e.*, $P(\mathbf{x}) = \{\mathbf{u} \in \mathbb{R}_+^M : \mathbf{x} \text{ can produce } \mathbf{u}\}$. It is also assumed that technology satisfies the usual axioms initially proposed by [Shephard \(1970\)](#), which allow to define the distance function -in terms of output- as the reciprocal of the maximum radial expansion of a given output vector proportional to the maximum output attainable. In such a way the resulting output

vector remains within $P(\mathbf{x})$, being attainable using the available resources and technology. The output distance can be formally defined as ¹:

$$D_o(\mathbf{x}, \mathbf{u}) = \inf \left\{ \theta : \left(\frac{\mathbf{u}}{\theta} \right) \in P(\mathbf{x}) \right\} \quad (1)$$

where $D_o(\mathbf{x}, \mathbf{u})$ is the distance from the banks output set to the frontier, and $\theta \in [0, 1]$ is the corresponding level of efficiency. The output distance function seeks the largest proportional increase in the observed output vector \mathbf{u} provided that the expanded vector $(\frac{\mathbf{u}}{\theta})$ is still an element of the original output set (Färe and Primont (1995)). Such an expression defines the *weak* disposability of outputs and therefore the inefficiency, which could explain, in our context, the presence of *NPLs* (undesirable outputs) that banks generate in their production processes, and that cannot freely eliminate either because it would require a greater use of inputs, and/or because resources would have to be diverted from marketable production.

In effect, by considering the *NPLs* as an output of a production process, other than to give the advantage of deriving the correspondent price, eliminates the empirical complications that would have occurred using a cost function approach. In fact, in this case a simultaneity problem would have been arisen between inefficiency and therefore costs- and *NPLs* considered as an explicative variable. Our approach exploits the duality of maximum revenue problem, expressed in terms of distance function (1), where the correspondence between the primal and the dual problems relies on efficiency and output prices. Furthermore, such an approach allows to define inefficiency as a function of outputs and prices, included that one of *NPLs*, on which the empirical analysis is focused. More specifically, undesirable outputs, such as *NPLs*, have non-positive shadow prices that may be obtained empirically by exploiting the above mentioned duality. Now we set primal and revenue function problems in order to find the two corresponding shadow prices vectors in natural numbers and normalized for the revenue function, respectively. Then, we find the *NPLs* price in natural numbers from the revenue function.

Denoting by $\mathbf{r} = (r_1, \dots, r_M)$ the output prices-vector, and assuming that $r_m \neq 0$, the revenue function in terms of the distance function may be expressed as:

$$\max R(\mathbf{x}, \mathbf{r}) = \max_u \left\{ \mathbf{r}' \mathbf{u} : \mathbf{u} \in P(\mathbf{x}) \right\} \quad (2)$$

¹This expression is equivalent to the reciprocal of the output oriented efficiency measure of Farrell (Farrell (1957) and Fare and Knox Lovell (1978)).

If the parent technology has convex output sets $P(\mathbf{x})$, for all $\mathbf{x} \in \mathbb{R}_+^N$, then one can prove (see [Shephard \(1970\)](#) or [Färe \(1988\)](#)) that the following duality holds:

$$\begin{aligned} R(\mathbf{x}, \mathbf{r}) &= \sup_u \{ \mathbf{r}'\mathbf{u} : D_o(\mathbf{x}, \mathbf{u}) \leq 1 \} \\ D_o(\mathbf{x}, \mathbf{u}) &= \sup_r \{ \mathbf{r}'\mathbf{u} : R(\mathbf{x}, \mathbf{r}) \leq 1 \} \end{aligned} \quad (3)$$

that is, the revenue function may be obtained by maximizing revenue with respect to outputs compatibly with the output distance function, which in its turn may be obtained by maximizing the actual revenue function with respect to output prices (normalized for the maximum revenue) compatibly with the attainable revenue out of the maximum one. Then, assuming that the revenue and distance functions are both differentiable, a Lagrange problem can be set up to maximize revenue:

$$\max_u \Lambda = \mathbf{r}'\mathbf{u} + \lambda (D_o(\mathbf{x}, \mathbf{u}) - 1) \quad (4)$$

and first order conditions with respect to outputs yield the relationship ([Färe and Primont \(1995\)](#)):

$$\mathbf{r} = -\lambda \nabla_u D_o(\mathbf{x}, \mathbf{u}) \quad (5)$$

At the optimum, in force of the homogeneity of degree 1 of $D_o(\mathbf{x}, \mathbf{u})$ (see [Jacobsen \(1972\)](#)), the negative of the Lagrange multiplier equals the revenue function, *i.e.*, $-\lambda = \Lambda = R(\mathbf{x}, \mathbf{r})$. Thus, we may write (5) in terms of the following system of equations:

$$\mathbf{r} = R(\mathbf{x}, \mathbf{r}) \nabla_u D_o(\mathbf{x}, \mathbf{u}) \quad (6)$$

Now by means of the second part of the duality theorem (3), we obtain that:

$$D_o(\mathbf{x}, \mathbf{u}) = r^*(\mathbf{x}, \mathbf{u}) \mathbf{u} \quad (7)$$

where $r^*(\mathbf{x}, \mathbf{u})$ represents the output price vector that maximises revenue.

Applying Shephard's dual lemma to expression (7), yields:

$$\nabla_u D_o(\mathbf{x}, \mathbf{u}) = r^*(\mathbf{x}, \mathbf{u}) \quad (8)$$

which, combined with (6), leads to:

$$\mathbf{r} = R(\mathbf{x}, \mathbf{r}) r^*(\mathbf{x}, \mathbf{u}) \quad (9)$$

where, $r^*(\mathbf{x}, \mathbf{u})$ is obtained from the gradient of the distance function, and represents revenue-deflated output prices. The main difficulty that arises in order to obtain absolute shadow prices from expression (9) is due to the dependence of the revenue function $R(\mathbf{x}, \mathbf{r})$ on \mathbf{r} , that is precisely the vector of shadow prices we are seeking for.

Therefore, in order to obtain $R(\mathbf{x}, \mathbf{r})$ we assume that "The observed price of an output m , r_m^o , equals its absolute shadow price r_m^* ", which allows to obtain the maximum revenue as:

$$R = \frac{r_m^o}{r_m^*(\mathbf{x}, \mathbf{u})} \quad (10)$$

which may be used to calculate the absolute shadow prices of the remaining outputs from its deflated shadow prices r^* . Denoting by $r_{m'}$ the absolute shadow prices for outputs other than m , we get:

$$r_{m'} = R \cdot r_{m'}^*(\mathbf{x}, \mathbf{u}) = R \cdot \frac{\partial D_o(\mathbf{x}, \mathbf{u})}{\partial u_{m'}} = r_m^o \cdot \frac{\partial D_o(\mathbf{x}, \mathbf{u}) / \partial u_{m'}}{\partial D_o(\mathbf{x}, \mathbf{u}) / \partial u_m} \quad (11)$$

3. Variables and data

Data are from Bankscope and are referred to 517 Commercial Banks in Europe and 2404 in the U.S., the sample period is 2000-2008. Europe includes the Euro system plus UK, Sweden, Norway and Turkey. The list of countries considered is reported in the following Tables 1 and 2. The large database used enables to asses very specifically both on the responsibilities of the single country policy and legislation and on the bank discipline during the last financial crisis.

The specification we adopt for the distance function is the *production approach* with three outputs and two inputs. Among *desirable* outputs we consider deposits (u_1), loans (u_2) and services (u_3), *NPL* (u_4) is the *undesirable* output and inputs are capital (x_1) and labor (x_2). Deposits are regarded as an output, rather than an input, for the diminishing importance of the corresponding interest rate still in the commercial banking system. All variables are expressed in nominal (dollar) values at constant prices (year 2000). The labor price is calculated as total personnel cost divided by the number of employees. Fixed assets have been transformed from the historical cost evaluation of balance sheet (International Accounting Standards 16) to current cost. As for capital price we estimate the following indirect function where the total capital is proxied²:

²We tried also direct functions both linear and logarithmic and other indirect functions with less qualitative results available upon request.

$$\log(CapitalCost_{kt}) = \sum_{k=1}^K d_{pck} \cdot \log(pc_{k0}) + \beta_1 \log\left(\frac{A_t + L_t}{2}\right) + \varepsilon_{kt} \quad (12)$$

for $k = 1, \dots, K, t = 2000, \dots, 2008$

where A_t stands for total assets, L_t for total liabilities, pc_k are estimated coefficients of the dummy variables d_{pck} representing the capital price for each branch and $\beta_1 \log\left(\frac{A_t + L_t}{2}\right)$ is the proxied total capital.

The services variable is constructed as the total value of "net" services.

Importantly, *NPLs* have different definitions across European countries and in the U.S.. In particular, the U.S. definition includes only the protested credits whilst a more prudential definition is adopted in Europe where are also considered the uncertain loans. We may now calculate a first indicator of the banking system risk consisting in the empirical *NPLs* failure probability for loans given by *NPLs* out of loans and reported in Figure 1.

Below are shown the descriptive statistics. We consider the mean and the standard deviation of the variables used in the estimation.

Table 1: Descriptive statistics: Europe (Time average data are expressed in millions of Dollars)

Country	Statistics	Deposits (u_1)	Loans (u_2)	Services (u_3)	Capital (x_1)	Labour (x_2)	Non Performing loans (u_4)
Austria	Mean	1747,04	1213,82	18,32	5985,93	267	12,09
	St. Dev.	5977,25	4553,54	32,11	18095,67	797	27,11
Belgium	Mean	2031,03	1306,25	20,79	4488,13	404	17,84
	St. Dev.	2170,53	1677,11	26,58	4974,75	407	15,84
Denmark	Mean	4661,29	4252,55	47,77	15587,65	885	27,34
	St. Dev.	17543,28	14334,46	151,75	55413,08	2459	60,31
Finland	Mean	21048,81	14555,98	238,64	67574,18	3183	91,98
	St. Dev.	23227,00	15318,52	294,19	87932,30	3372	68,33
France	Mean	13656,28	6885,30	164,02	37467,77	1777	37,99
	St. Dev.	68586,04	30746,82	799,17	205861,30	6327	88,48
Germany	Mean	8495,53	5686,36	64,33	21259,51	815	30,50
	St. Dev.	43770,04	25874,97	331,86	107016,10	3456	81,58
Great Britain	Mean	1425,02	661,53	18,11	3747,84	134	9,62
	St. Dev.	3006,67	1646,89	41,26	9392,71	209	14,44
Greece	Mean	20988,58	14953,53	225,88	47050,90	5307	95,79
	St. Dev.	19250,46	13110,13	247,78	43570,73	4192	61,60
Ireland	Mean	1628,32	876,27	4,16	11277,91	28	14,63
	St. Dev.	1351,37	752,11	13,56	18714,80	24	9,34
Italy	Mean	9368,70	8047,37	151,08	29230,47	2156	51,09
	St. Dev.	25245,42	21729,86	405,87	83840,79	5555	76,63
Luxembourg	Mean	6482,70	1791,80	51,02	14633,63	253	21,08
	St. Dev.	9543,99	2645,64	86,90	20937,75	429	21,58

Continued on Next Page.

Table 1: Descriptive statistics: Europe (Time average data are expressed in millions of Dollars)

Country	Statistics	Deposits (u_1)	Loans (u_2)	Services (u_3)	Capital (x_1)	Labour (x_2)	Non Performing loans (u_4)
Norway	Mean	8917,08	8516,49	91,95	20680,95	1067	57,94
	St. Dev.	12749,99	11497,77	127,96	28475,77	1309	62,61
Holland	Mean	1739,90	961,61	19,54	3634,53	181	16,90
	St. Dev.	573,13	327,12	17,51	1263,83	181	3,88
Portugal	Mean	12354,38	9385,98	183,55	32140,49	1924	58,88
	St. Dev.	17147,59	13751,15	214,46	45620,39	2535	71,89
Spain	Mean	28746,29	21868,65	305,62	73064,57	6056	107,96
	St. Dev.	50308,91	35403,95	654,43	140552,40	12693	120,13
Sweden	Mean	12186,94	6861,50	123,59	31601,42	1401	39,05
	St. Dev.	28378,89	16713,39	290,24	74676,69	3093	75,51
Switzerland	Mean	8456,13	3261,29	150,64	19532,63	677	17,27
	St. Dev.	65926,16	22076,50	1158,58	151058,40	4533	66,77
Turkey	Mean	6663,87	3679,48	177,47	15626,92	6039	33,50
	St. Dev.	7067,57	4082,52	197,98	16380,99	7292	34,09
Europe	Mean	8864,66	5420,21	105,41	23524,42	1246	32,79
	St. Dev.	43645,11	21728,41	588,05	116975,50	4685	72,43

Table 2: Descriptive statistics: U.S. (Time average data are expressed in millions of Dollars)

Country	Statistics	Deposits (u_1)	Loans (u_2)	Services (u_3)	Capital (x_1)	Labour (x_2)	Non Performing loans (u_4)
Alabama	Mean	138,33	105,40	1,28	461,65	60	1,46
	St. Dev.	153,33	126,36	1,71	509,40	69	2,05
Alaska	Mean	1069,90	744,94	17,38	3502,90	484	9,98
	St. Dev.	457,51	219,85	12,38	1661,77	225	2,51
Arizona	Mean	104,74	93,13	0,67	332,77	35	1,26
	St. Dev.	54,12	45,70	0,55	175,67	21	1,07
Arkansas	Mean	216,39	165,79	2,69	698,86	99	2,40
	St. Dev.	277,49	219,59	4,65	893,72	121	3,19
California	Mean	317,58	274,19	3,38	1050,45	107	3,73
	St. Dev.	429,59	424,74	5,87	1487,87	134	5,45
Colorado	Mean	291,00	227,30	3,38	910,52	99	2,96
	St. Dev.	368,11	313,67	4,69	1131,38	107	4,50
Connecticut	Mean	359,14	337,63	1,90	1224,77	105	3,25
	St. Dev.	216,80	236,87	2,30	770,70	54	2,16
Delaware	Mean	572,83	440,43	17,46	1859,01	138	7,05
	St. Dev.	795,49	601,74	30,20	2570,17	131	10,10
Florida	Mean	219,11	179,88	2,28	704,21	80	2,35
	St. Dev.	274,42	239,94	5,50	887,63	112	3,25
Georgia	Mean	162,21	134,68	1,65	519,78	64	2,01
	St. Dev.	159,96	142,80	2,46	513,98	64	2,41
Idaho	Mean	259,08	216,61	3,64	827,18	143	3,47
	St. Dev.	199,31	179,38	3,34	631,46	102	3,07
Illinois	Mean	250,90	191,98	2,69	806,28	87	2,46
	St. Dev.	535,00	396,56	7,18	1728,90	169	6,08
Indiana	Mean	302,54	252,21	4,19	1009,05	127	2,90
	St. Dev.	327,11	276,36	8,03	1093,48	142	3,21
Iowa	Mean	122,67	103,12	1,15	410,98	45	2,46
	St. Dev.	146,09	134,27	1,76	509,43	49	10,93

Continued on Next Page.

Table 2: Descriptive statistics: U.S. (Time average data are expressed in millions of Dollars)

Country	Statistics	Deposits (u_1)	Loans (u_2)	Services (u_3)	Capital (x_1)	Labour (x_2)	Non Performing loans (u_4)
<i>Kansas</i>	Mean	111,72	88,82	1,75	364,69	52	1,23
	St. Dev.	128,16	105,72	4,76	423,43	64	1,62
<i>Kentucky</i>	Mean	156,75	130,23	1,84	514,18	68	1,68
	St. Dev.	175,14	156,02	3,35	559,22	68	2,06
<i>Louisiana</i>	Mean	166,92	122,56	2,14	530,11	92	1,80
	St. Dev.	137,55	109,59	2,13	432,44	77	2,43
<i>Maine</i>	Mean	357,92	332,59	3,99	1239,64	133	3,99
	St. Dev.	218,25	184,48	4,15	752,53	47	2,63
<i>Maryland</i>	Mean	256,04	225,10	2,50	843,20	121	2,63
	St. Dev.	253,44	220,72	3,81	851,96	148	2,58
<i>Massachusetts</i>	Mean	170,91	140,21	1,38	572,73	59	1,60
	St. Dev.	228,99	184,21	2,72	791,37	73	2,49
<i>Michigan</i>	Mean	232,37	209,43	2,26	771,86	91	3,01
	St. Dev.	323,53	294,21	3,85	1081,65	135	4,49
<i>Minnesota</i>	Mean	168,90	142,17	1,46	544,25	55	1,92
	St. Dev.	226,04	184,37	2,14	732,36	52	2,69
<i>Mississippi</i>	Mean	207,18	158,74	2,65	671,60	99	2,24
	St. Dev.	227,22	195,03	4,30	742,07	118	2,54
<i>Missouri</i>	Mean	167,02	136,90	1,94	536,07	70	1,86
	St. Dev.	212,98	172,58	5,02	689,62	85	2,80
<i>Montana</i>	Mean	152,25	126,38	1,44	501,07	64	1,88
	St. Dev.	141,65	124,18	1,69	469,14	51	2,00
<i>Nebraska</i>	Mean	157,92	134,32	1,70	507,59	65	2,09
	St. Dev.	234,49	207,96	3,77	748,72	90	3,83
<i>Nevada</i>	Mean	296,77	232,23	0,09	930,84	81	3,57
	St. Dev.	450,03	367,39	8,84	1412,74	97	6,81
<i>New Hampshire</i>	Mean	278,76	236,47	2,15	923,97	114	2,89
	St. Dev.	180,83	148,04	1,83	593,58	75	2,46
<i>New Jersey</i>	Mean	403,32	291,87	2,06	1311,36	103	3,86
	St. Dev.	722,25	431,84	5,49	2398,59	195	7,58
<i>New Mexico</i>	Mean	177,40	135,37	1,77	568,59	75	1,79
	St. Dev.	211,24	190,84	2,11	684,07	65	2,12
<i>New York</i>	Mean	503,54	302,44	4,61	1653,79	135	4,19
	St. Dev.	938,82	420,23	8,81	3146,15	153	7,61
<i>North Carolina</i>	Mean	388,06	340,69	3,74	1277,65	149	4,70
	St. Dev.	316,63	286,39	3,73	1022,51	127	4,33
<i>North Dakota</i>	Mean	170,11	145,40	2,10	545,12	76	2,04
	St. Dev.	248,47	234,69	4,28	794,56	107	3,21
<i>Ohio</i>	Mean	155,69	124,61	1,47	518,19	68	1,45
	St. Dev.	202,46	161,72	3,26	706,08	86	2,24
<i>Oklahoma</i>	Mean	144,26	113,47	1,91	461,54	73	1,31
	St. Dev.	167,05	140,41	2,70	537,30	78	1,76
<i>Oregon</i>	Mean	298,07	272,26	3,35	951,79	147	3,82
	St. Dev.	298,40	302,82	3,34	956,51	127	5,42
<i>Pennsylvania</i>	Mean	306,48	238,52	2,95	1040,64	121	2,66
	St. Dev.	244,16	190,12	3,81	823,73	112	2,24
<i>Rhode Island</i>	Mean	844,72	740,07	7,07	3027,55	245	10,01
	St. Dev.	121,37	130,17	1,71	615,99	29	1,39
<i>South Carolina</i>	Mean	216,28	185,88	2,37	716,97	88	2,32
	St. Dev.	220,79	211,66	3,45	712,27	95	2,80
<i>South Dakota</i>	Mean	215,88	179,46	3,33	689,67	88	2,11
	St. Dev.	354,76	300,89	11,95	1102,97	122	3,30

Continued on Next Page.

Table 2: Descriptive statistics: U.S. (Time average data are expressed in millions of Dollars)

Country	Statistics	Deposits (u_1)	Loans (u_2)	Services (u_3)	Capital (x_1)	Labour (x_2)	Non Performing loans (u_4)
Tennessee	Mean	173,61	140,24	1,77	554,16	74	5,09
	St. Dev.	202,39	176,61	2,14	646,64	63	26,92
Texas	Mean	255,25	177,31	4,20	809,56	115	2,18
	St. Dev.	559,52	427,80	11,81	1813,10	217	4,98
Utah	Mean	223,56	187,50	3,01	713,14	99	2,95
	St. Dev.	178,44	153,63	3,03	557,56	87	3,58
Vermont	Mean	200,52	163,03	2,18	644,51	102	1,94
	St. Dev.	73,33	67,46	1,53	259,36	34	0,95
Virginia	Mean	243,88	199,92	2,94	790,76	111	2,48
	St. Dev.	197,69	169,57	4,81	664,15	90	2,43
Washington	Mean	275,15	233,72	3,37	898,31	111	3,24
	St. Dev.	401,96	368,43	6,59	1317,89	151	5,52
West Virginia	Mean	230,27	187,14	2,14	761,62	101	2,25
	St. Dev.	507,27	415,63	6,14	1764,97	219	5,21
Wisconsin	Mean	243,59	204,84	2,54	794,74	77	2,73
	St. Dev.	1095,49	900,07	16,11	3614,55	309	12,84
Wyoming	Mean	117,28	79,66	0,94	363,87	45	0,82
	St. Dev.	85,91	60,73	1,10	265,02	37	0,49
U.S.	Mean	219,53	174,83	2,44	714,60	85	2,43
	St. Dev.	426,82	332,39	6,68	1405,78	137	6,91

Figure 1, coherently with the wider definition of non performing loans in Europe, shows that the ratio $NPLs/LOANS$ is always higher in Europe with an average of 2.7% compared with 1,4% of the U.S.. We note however that in Europe the ratio decreases during the years considered in contrast to the U.S. where it increases especially in 2008, raised from 1,36% at 1,57%.

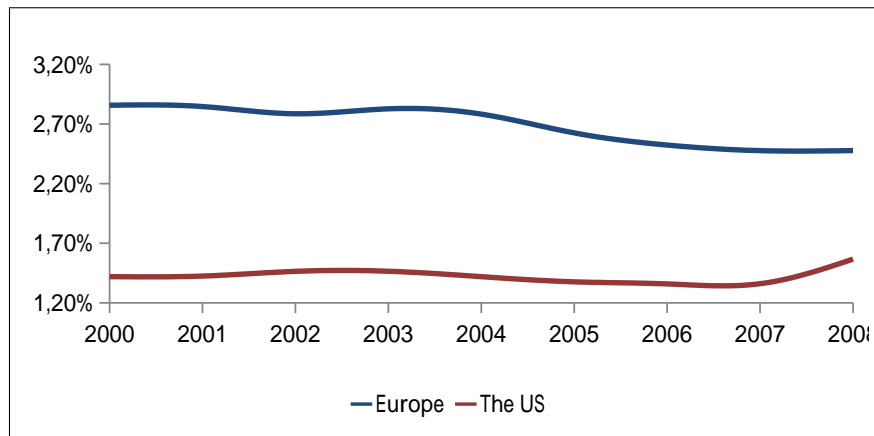


Figure 1: NPLs/Loans series of Europe and the U.S. (years 2000-2008)

4. Empirical methodology

In order to be able to calculate the shadow prices of the *NPLs* as described in section 2, in this section we estimate with a *Feasible Generalized Least Squares regression (FGLS)* the distance function. Following [Aigner and Chu \(1968\)](#) the problem to be solved is:

$$\max \sum_{k=1}^K \left[\log D_o(x^k, u^k) - \log(1) \right] \quad (13)$$

where $k = 1, \dots, K$ indexes individual banks.

This function is subject to the following constraints:

- (i) $\log D_o(x^k, u^k) \leq 0, k = 1, \dots, K$
- (ii) $\frac{\partial \log D_o(x^k, u^k)}{\partial \log u_m^k} \geq 0, m = 1, \dots, h; k = 1, \dots, K$
- (iii) $\frac{\partial \log D_o(x^k, u^k)}{\partial \log u_m^k} \leq 0, m = h + 1, \dots, M; k = 1, \dots, K$
- (iv) $\sum_{m=1}^M \alpha_m = 1, \sum_{m=1}^M \alpha_{mm'} = \sum_{m=1}^M \gamma_{nm} = 0,$
- (v) $\beta_{nn'} = \beta_{n'n}, \alpha_{mm'} = \alpha_{m'm}, \gamma_{nm} = \gamma_{mn}, \delta_{ij} = \delta_{ji}, \lambda_{ij} = \lambda_{ji},$
 $m = 1, \dots, M, n = 1, \dots, N, i, j = 1, \dots, N + M.$

where the first h outputs are desirables and the next $(M-h)$ outputs are undesirables.

The objective function "minimizes" the sum of deviations of individual observations from the frontier of technology. The set of restrictions in (i) implies that each observation is located either on or below the technological frontier; the restrictions contained in (ii) ensure that desirable outputs will have nonnegative shadow prices for all firms, while (iii) undesirable outputs will have nonpositive shadow prices, also for all firms. The assumption of weak disposal of outputs is introduced by restriction (iv) that imposes homogeneity of degree 1 in outputs; finally (v) imposes symmetry.

The specification we adopt is the Fourier Flexible Functional form (FFF) which can globally approximate the unknown true function³. In order to test

³The FFF, developed by [Gallant \(1981\)](#), combines the standard TL with the non-parametric

the robustness of our results, we also estimate the Translog (TL) being the most broadly used flexible functional form⁴.

The FFF can be expressed as follows:

$$\begin{aligned}
\ln D_o = & \alpha_0 + \sum_{n=1}^N \beta_n \cdot \ln x_n + \sum_{m=1}^M \alpha_m \cdot \ln u_m + \frac{1}{2} \sum_{n=1}^N \sum_{n'=1}^N \beta_{nn'} \cdot (\ln x_n) \cdot (\ln x_{n'}) \\
& + \frac{1}{2} \sum_{m=1}^M \sum_{m'=1}^M \alpha_{mm'} \cdot (\ln u_m) \cdot (\ln u_{m'}) + \sum_{n=1}^N \sum_{m=1}^M \gamma_{nm} \cdot (\ln x_n) \cdot (\ln u_m) \\
& + \sum_{i=1}^{M+N} \delta_i \cdot \sin(z_i) + \sum_{i=1}^{M+N} \lambda_i \cdot \cos(z_i) + \sum_{i=1}^{M+N} \sum_{j=1}^{M+N} \delta_{ij} \cdot \sin(z_i + z_j) \\
& + \sum_{i=1}^{M+N} \sum_{j=1}^{M+N} \lambda_{ij} \cdot \cos(z_i + z_j) + \sum_{i=1}^{M+N} \sum_{j=1}^{M+N} \sum_{l=1}^{M+N} \delta_{ijl} \cdot \sin(z_i + z_j + z_l) \\
& + \sum_{i=1}^{M+N} \sum_{j=1}^{M+N} \sum_{l=1}^{M+N} \lambda_{ijl} \cdot \cos(z_i + z_j + z_l) + \varepsilon
\end{aligned} \tag{14}$$

As for the determination of the frontier, D_o needs to be equal to unity and, in that case, the logarithm of the term on the left side of the equation (14) will equate zero. Consequently, it is necessary that outputs meet the homogeneity condition of degree 1 in order to satisfy the restriction (iv). Following [Lovell et al. \(1994\)](#), this condition has been imposed by normalising the distance function with one of the outputs. This starts from the assumption that homogeneity implies that:

$$D_o \left(\mathbf{x}, \frac{\mathbf{u}}{u_M} \right) = \frac{D_o(\mathbf{x}, \mathbf{u})}{u_M} \tag{15}$$

Substituting $u_m^* = \frac{u_m}{u_M}, m = 1, \dots, M-1$ in (14) we obtain a regression of the general form:

$$\ln(D_o/u_M) = FFF(\mathbf{x}, \mathbf{u}^*, \alpha, \beta, \gamma, \lambda, \delta) \tag{16}$$

Fourier form. The number of trigonometric terms in the FFF has been chosen, following the rule of thumb expounded in [Eastwood and Gallant \(1991\)](#) to get a total number of parameters equal to the number of the observations raised to the power of two-thirds. Such a rule serves to obtain consistent and asymptotically normal estimates. However, as suggested in [Gallant \(1981\)](#), the effective number of coefficients may be corrected, by reducing the number of trigonometric terms, to avoid possible multicollinearity consequences.

⁴Results available upon request.

where $\mathbf{u}^* = (\frac{u_1}{u_M}, \frac{u_2}{u_M}, \dots, \frac{u_{M-1}}{u_M})$.
Equation (16) can be written as:

$$-ln(u_M) = FFF(\mathbf{x}, \mathbf{u}^*, \alpha, \beta, \gamma, \lambda, \delta) - ln(D_o) \quad (17)$$

In equation (17) the $-ln(D_o)$ can be interpreted as an error term which captures the technical inefficiency.

Finally, in order to improve the quality of the FFF approximation, and to have a reference with the Taylor expansion, outputs (\mathbf{u}) and inputs (\mathbf{x}) are all expressed as differences from the sample mean.

Therefore, the estimated FFF is:

$$\begin{aligned} -lnu_M = & \alpha_0 + \sum_{n=1}^N \beta_n \cdot lnx_n + \sum_{m=1}^{M-1} \alpha_m \cdot lnu_m^* + \frac{1}{2} \sum_{n=1}^N \sum_{n'=1}^N \beta_{nn'} \cdot (lnx_n) \cdot (lnx_{n'}) \\ & + \frac{1}{2} \sum_{m=1}^{M-1} \sum_{m'=1}^{M-1} \alpha_{mm'} \cdot (lnu_m^*) \cdot (lnu_{m'}^*) + \sum_{n=1}^N \sum_{m=1}^{M-1} \gamma_{nm} \cdot (lnx_n) \cdot (lnu_m^*) \\ & + \sum_{i=1}^{M-1+N} \delta_i \cdot \sin(z_i) + \sum_{i=1}^{M-1+N} \lambda_i \cdot \cos(z_i) + \sum_{i=1}^{M-1+N} \sum_{j=1}^{M-1+N} \delta_{ij} \cdot \sin(z_i + z_j) \\ & + \sum_{i=1}^{M-1+N} \sum_{j=1}^{M-1+N} \lambda_{ij} \cdot \cos(z_i + z_j) + \sum_{i=1}^{M-1+N} \sum_{j=1}^{M-1+N} \sum_{l=1}^{M-1+N} \delta_{ijl} \cdot \sin(z_i + z_j + z_l) \\ & + \sum_{i=1}^{M-1+N} \sum_{j=1}^{M-1+N} \sum_{l=1}^{M-1+N} \lambda_{ijl} \cdot \cos(z_i + z_j + z_l) + \varepsilon \end{aligned} \quad (18)$$

where $u_m^* = \frac{u_m}{u_M}, m = 1, \dots, M-1$ and $\varepsilon = -ln(D_o) + ln(v)$.

For coherency purposes we have transformed the original independent variables in radians to be used in the trigonometric part of the function as in [Berger et al. \(1997\)](#): $z_i = 0.2 \cdot \pi - \mu \cdot a + \mu \cdot ln(y_i)$ where $\mu \equiv \frac{0.9 \cdot 2\pi - 0.1 \cdot 2\pi}{(b-a)}$ and $[a, b]$ is the range of $ln(y_i)$. In this case $ln(y_i)$ with $i = 1, \dots, 6$ refers to the sequence of deposits, loans, services, *NPLs*, capital and labor.

Once estimated the distance function, we calculate the efficiency by adopting the "Free efficiency" method (see [Berger \(1993\)](#)):

$$TE_k = \exp \left\{ - \left[\max_k (\widehat{\varepsilon}_{.k}) - \widehat{\varepsilon}_{.k} \right] \right\} \quad (19)$$

where $\widehat{\varepsilon}_{.k} = \sum_t \varepsilon_{tk}/T$.

Then the shadow price of *NPLs* may be found according to the procedure expounded above.

Hence, we estimate the price of loans by assuming that its shadow price is equal to its market price. So, we compute normalized shadow prices $r^*(\mathbf{x}, \mathbf{u})$ of desirable and undesirable outputs for each bank, using (8), and we calculate the shadow revenue R using the (10). Given the shadow revenue, we derive absolute shadow prices for *NPLs* using the (11).

5. Estimation

In this section we report the results of our estimation. All variables have been divided by its sample mean so that the first-order coefficients can be interpreted as distance elasticities evaluated at the sample means. The linear homogeneity in outputs is imposed using the output "Deposits" as a numeraire⁵. Due to multicollinearity we consider the Fourier approximation till the third term and drop some of the regressors^{6 7}.

5.1. Europe

Table 3: Distance function estimation: Europe

Dependent variable:	Coef.	Std. Err.	z	$P > z$	95% Conf.	Interval
$\ln(1/u_1)$						
$\ln(u_2/u_1)$	0.349	0.113	3.080	0.002	0.127	0.572
$\ln(u_3/u_1)$	0.153	0.049	3.120	0.002	0.057	0.249
$\ln(u_4/u_1)$	-0.352	0.204	-1.730	0.084	-0.751	0.047
$\ln(x_1)$	-0.466	0.064	-7.240	0.000	-0.593	-0.340
$\ln(x_2)$	-0.771	0.033	-23.030	0.000	-0.836	-0.705
$\ln(u_2/u_1)^2$	0.084	0.011	7.810	0.000	0.063	0.105
$\ln(u_2/u_1)\ln(u_3/u_1)$	-0.050	0.006	-8.360	0.000	-0.062	-0.039
$\ln(u_2/u_1)\ln(u_4/u_1)$	0.017	0.003	4.970	0.000	0.010	0.024
$\ln(u_3/u_1)^2$	-0.007	0.016	-0.470	0.640	-0.039	0.024
$\ln(u_3/u_1)\ln(u_4/u_1)$	0.008	0.002	3.760	0.000	0.004	0.012
$\ln(u_4/u_1)^2$	-0.052	0.020	-2.630	0.009	-0.092	-0.013
$\ln(x_1)^2$	-0.029	0.017	-1.690	0.092	-0.062	0.005
$\ln(x_1)\ln(x_2)$	-0.043	0.006	-7.730	0.000	-0.054	-0.032
$\ln(x_2)^2$	-0.106	0.020	-5.180	0.000	-0.146	-0.066
$\ln(u_2/u_1)\ln(x_1)$	-0.017	0.008	-1.970	0.048	-0.033	0.000

Continued on Next Page.

⁵The choice of the output is arbitrary and the resulting estimates are invariant to the normalization (see Cuesta and Orea (2002)).

⁶All estimations and calculations have been done with Stata 11 software.

⁷Please see section 3 at page 6 for the meaning of the variables notations.

Table 3: Distance function estimation: Europe

Dependent variable:	Coef.	Std. Err.	z	$P > z$	95% Conf.	Interval
$\ln(1/u_1)$						
$\ln(u_2/u_1)\ln(x_2)$	0.011	0.009	1.180	0.236	-0.007	0.029
$\ln(u_3/u_1)\ln(x_1)$	-0.049	0.015	-3.370	0.001	-0.078	-0.021
$\ln(u_3/u_1)\ln(x_2)$	-0.048	0.012	-3.940	0.000	-0.072	-0.024
$\ln(u_4/u_1)\ln(x_1)$	0.005	0.003	1.980	0.047	0.000	0.010
$\ln(u_4/u_1)\ln(x_2)$	-0.029	0.003	-10.350	0.000	-0.034	-0.023
$\sin(z_2)$	-0.029	0.549	-0.050	0.957	-1.106	1.047
$\sin(z_4)$	-1.879	1.011	-1.860	0.063	-3.860	0.102
$\sin(z_5)$	-0.054	0.272	-0.200	0.844	-0.586	0.479
$\cos(z_{22})$	-0.124	0.086	-1.440	0.149	-0.292	0.044
$\sin(z_{22})$	-0.526	0.209	-2.520	0.012	-0.935	-0.116
$\cos(z_{33})$	0.326	0.059	5.490	0.000	0.210	0.443
$\sin(z_{33})$	-0.465	0.042	-10.960	0.000	-0.548	-0.382
$\cos(z_{44})$	-0.244	0.152	-1.610	0.108	-0.542	0.053
$\sin(z_{44})$	-0.515	0.264	-1.950	0.051	-1.033	0.003
$\cos(z_{55})$	0.127	0.039	3.230	0.001	0.050	0.204
$\sin(z_{55})$	-0.224	0.090	-2.490	0.013	-0.401	-0.048
$\cos(z_{66})$	0.092	0.077	1.200	0.231	-0.058	0.242
$\sin(z_{66})$	-0.212	0.068	-3.130	0.002	-0.344	-0.079
$\cos(z_{23})$	-0.176	0.043	-4.110	0.000	-0.259	-0.092
$\sin(z_{23})$	0.251	0.023	10.840	0.000	0.206	0.297
$\sin(z_{24})$	-0.121	0.030	-4.040	0.000	-0.180	-0.062
$\cos(z_{25})$	-0.095	0.043	-2.220	0.026	-0.180	-0.011
$\sin(z_{25})$	0.036	0.024	1.500	0.135	-0.011	0.082
$\cos(z_{26})$	0.005	0.042	0.130	0.898	-0.076	0.087
$\sin(z_{26})$	-0.054	0.027	-2.010	0.045	-0.108	-0.001
$\cos(z_{35})$	-0.271	0.074	-3.650	0.000	-0.417	-0.125
$\sin(z_{35})$	-0.270	0.042	-6.420	0.000	-0.353	-0.188
$\cos(z_{36})$	-0.050	0.059	-0.850	0.396	-0.165	0.065
$\sin(z_{36})$	-0.256	0.035	-7.380	0.000	-0.324	-0.188
$\cos(z_{56})$	-0.132	0.022	-5.880	0.000	-0.176	-0.088
$\sin(z_{56})$	0.032	0.018	1.760	0.079	-0.004	0.069
$\cos(z_{222})$	-0.099	0.039	-2.540	0.011	-0.176	-0.023
$\cos(z_{333})$	0.260	0.030	8.610	0.000	0.201	0.319
$\cos(z_{444})$	-0.115	0.051	-2.250	0.024	-0.215	-0.015
$\cos(z_{555})$	-0.010	0.016	-0.640	0.524	-0.043	0.022
$\cos(z_{666})$	0.078	0.023	3.410	0.001	0.033	0.123
$\sin(z_{222})$	-0.175	0.060	-2.940	0.003	-0.292	-0.058
$\sin(z_{333})$	-0.079	0.021	-3.770	0.000	-0.120	-0.038
$\sin(z_{444})$	-0.088	0.054	-1.620	0.106	-0.194	0.018
$\sin(z_{555})$	-0.017	0.026	-0.650	0.517	-0.069	0.035
$\sin(z_{666})$	-0.132	0.030	-4.450	0.000	-0.190	-0.074
t	-0.005	0.003	-1.670	0.096	-0.011	0.001
$t(u_2/u_1)$	-0.007	0.002	-4.700	0.000	-0.010	-0.004
$t(u_3/u_1)$	-0.001	0.001	-0.450	0.652	-0.003	0.002
$t(u_4/u_1)$	0.005	0.001	4.940	0.000	0.003	0.006
$t(x_1)$	0.006	0.001	4.110	0.000	0.003	0.008
$t(x_2)$	-0.003	0.001	-1.870	0.061	-0.006	0.000
<i>Austria</i>	-0.928	0.150	-6.180	0.000	-1.223	-0.634
<i>Belgium</i>	-1.019	0.151	-6.730	0.000	-1.316	-0.722
<i>Denmark</i>	-0.756	0.150	-5.050	0.000	-1.050	-0.463
<i>Finland</i>	-1.031	0.158	-6.540	0.000	-1.340	-0.722
<i>France</i>	-0.988	0.150	-6.600	0.000	-1.282	-0.695
<i>Germany</i>	-1.108	0.150	-7.390	0.000	-1.402	-0.814
<i>Greece</i>	-0.718	0.153	-4.690	0.000	-1.018	-0.418

Continued on Next Page.

Table 3: Distance function estimation: Europe

Dependent variable: $\ln(1/u_1)$	Coef.	Std. Err.	z	$P > z$	95% Conf.	Interval
<i>Great Britain</i>	-0.954	0.151	-6.320	0.000	-1.251	-0.658
<i>Ireland</i>	-1.496	0.173	-8.650	0.000	-1.835	-1.157
<i>Italy</i>	-1.042	0.150	-6.950	0.000	-1.336	-0.748
<i>Luxembourg</i>	-1.443	0.151	-9.560	0.000	-1.739	-1.147
<i>Netherlands</i>	-0.641	0.163	-3.920	0.000	-0.961	-0.321
<i>Norway</i>	-1.285	0.160	-8.030	0.000	-1.598	-0.971
<i>Portugal</i>	-0.844	0.159	-5.320	0.000	-1.155	-0.533
<i>Spain</i>	-1.038	0.152	-6.850	0.000	-1.336	-0.741
<i>Sweden</i>	-1.136	0.151	-7.540	0.000	-1.431	-0.841
<i>Switzerland</i>	-1.151	0.150	-7.660	0.000	-1.446	-0.857
<i>Turkey</i>	(omitted)					
<i>Constant</i>	2.570	0.303	8.490	0.000	1.976	3.163

We get estimates significant and coherent with the literature (see among others [Cuesta and Orea \(2002\)](#)). In particular, looking at the elasticities of the first order terms, we find positive coefficients for desirable outputs (Loans, deposits and services)⁸ and negative for the undesirable output (*NPLs*). The negative sign of the latter represents the opportunity cost measurable in terms of the loss in desirable outputs production that banks would incur in case of compliance with a regulation directed to compensate the *NPLs*.

As regards the management of inputs, the labor factor has a greater impact (0.771) on the production possibilities frontier than capital (0.466).

Looking at the country dummies, we can identify the spatial effect on efficiency (with respect to Turkey, the reference state). These differences can be caused by factors not considered in the analysis such as technology, environmental factors, externalities, etc.

The dummies are all negative, which means that they increase -other things being equal- the efficiency with respect to the base country (Turkey).

From this analysis it would seem that Ireland is the most efficient country compared to Turkey, and that the most inefficient are Netherlands, Greece and Denmark. These results seem implausible in light of the past financial crisis. Hence, we think that the distance function lacks to considering, among inputs, the risk arising from the amount of *NPLs*.

Finally, the time variable (trend) has a negative coefficient (-0.01), which suggests that over time, there was on average an efficiency decrease of 1%.

⁸Remember that the function is homogeneous of degree 1 in outputs, so that the coefficient of deposits is given by $1 - \sum_{m=1}^{M-1} \alpha_m$.

5.2. The U.S.

Table 4: Distance function estimation: U.S.

Dependent variable: $\ln(1/u_1)$	Coef.	Std. Err.	z	$P > z$	95% Conf.	Interval
$\ln(u_2/u_1)$	0.302	0.043	7.040	0.000	0.218	0.386
$\ln(u_3/u_1)$	0.165	0.024	6.800	0.000	0.117	0.212
$\ln(u_4/u_1)$	-0.275	0.124	-2.210	0.027	-0.518	-0.032
$\ln(x_1)$	-0.215	0.023	-9.380	0.000	-0.260	-0.170
$\ln(x_2)$	-0.862	0.023	-38.100	0.000	-0.907	-0.818
$\ln(u_2/u_1)\ln(u_3/u_1)$	0.012	0.009	1.330	0.185	-0.006	0.029
$\ln(u_2/u_1)\ln(u_4/u_1)$	-0.060	0.009	-6.360	0.000	-0.078	-0.041
$\ln(u_3/u_1)^2$	-0.087	0.017	-5.210	0.000	-0.120	-0.055
$\ln(u_3/u_1)\ln(u_4/u_1)$	-0.010	0.005	-1.800	0.072	-0.020	0.001
$\ln(x_1)^2$	0.028	0.016	1.820	0.069	-0.002	0.059
$\ln(x_1)\ln(x_2)$	-0.137	0.005	-29.180	0.000	-0.147	-0.128
$\ln(x_2)^2$	-0.118	0.024	-4.920	0.000	-0.166	-0.071
$\ln(u_2/u_1)\ln(x_1)$	0.217	0.020	10.970	0.000	0.179	0.256
$\ln(u_2/u_1)\ln(x_2)$	-0.203	0.034	-5.910	0.000	-0.270	-0.136
$\ln(u_3/u_1)\ln(x_1)$	-0.078	0.016	-4.970	0.000	-0.109	-0.047
$\ln(u_3/u_1)\ln(x_2)$	-0.123	0.004	-28.490	0.000	-0.132	-0.115
$\ln(u_4/u_1)\ln(x_1)$	-0.032	0.003	-11.820	0.000	-0.038	-0.027
$\ln(u_4/u_1)\ln(x_2)$	-0.040	0.003	-11.320	0.000	-0.046	-0.033
$\cos(z_2)$	0.436	0.256	1.710	0.088	-0.065	0.937
$\sin(z_4)$	-0.811	0.385	-2.100	0.035	-1.566	-0.055
$\cos(z_{22})$	0.094	0.143	0.660	0.510	-0.186	0.374
$\sin(z_{22})$	-0.033	0.021	-1.580	0.115	-0.074	0.008
$\sin(z_{33})$	-0.214	0.022	-9.650	0.000	-0.257	-0.171
$\cos(z_{44})$	0.169	0.016	10.600	0.000	0.138	0.200
$\sin(z_{44})$	-0.199	0.112	-1.780	0.075	-0.417	0.020
$\cos(z_{55})$	0.104	0.023	4.410	0.000	0.058	0.150
$\sin(z_{55})$	0.201	0.015	13.210	0.000	0.171	0.231
$\cos(z_{66})$	-0.476	0.035	-13.450	0.000	-0.546	-0.407
$\sin(z_{66})$	0.061	0.043	1.440	0.150	-0.022	0.145
$\cos(z_{25})$	0.279	0.025	10.970	0.000	0.229	0.329
$\sin(z_{25})$	-0.126	0.015	-8.210	0.000	-0.156	-0.096
$\cos(z_{26})$	-0.245	0.033	-7.530	0.000	-0.309	-0.181
$\sin(z_{26})$	0.116	0.027	4.340	0.000	0.063	0.168
$\cos(z_{34})$	-0.025	0.013	-1.910	0.057	-0.051	0.001
$\sin(z_{34})$	0.008	0.009	0.940	0.345	-0.009	0.025
$\cos(z_{35})$	-0.431	0.042	-10.180	0.000	-0.514	-0.348
$\sin(z_{35})$	-0.136	0.013	-10.270	0.000	-0.162	-0.110
$\cos(z_{222})$	-0.064	0.042	-1.520	0.128	-0.145	0.018
$\cos(z_{333})$	-0.041	0.005	-7.910	0.000	-0.051	-0.031
$\cos(z_{444})$	0.052	0.007	7.320	0.000	0.038	0.066
$\cos(z_{555})$	0.018	0.009	2.050	0.040	0.001	0.036
$\cos(z_{666})$	-0.125	0.010	-12.550	0.000	-0.144	-0.105
$\sin(z_{333})$	-0.121	0.011	-11.020	0.000	-0.142	-0.099
$\sin(z_{444})$	-0.013	0.024	-0.550	0.582	-0.060	0.034
$\sin(z_{555})$	0.038	0.007	5.680	0.000	0.025	0.051
$\sin(z_{666})$	0.115	0.017	6.870	0.000	0.082	0.148
t	0.013	0.001	9.860	0.000	0.010	0.015
$t(u_2/u_1)$	0.008	0.002	3.650	0.000	0.004	0.013
$t(u_3/u_1)$	0.008	0.001	8.700	0.000	0.006	0.010
$ty(u_4/u_1)$	-0.002	0.001	-3.380	0.001	-0.004	-0.001
$t(x_1)$	0.014	0.001	16.570	0.000	0.012	0.015
$t(x_2)$	-0.011	0.001	-11.030	0.000	-0.013	-0.009

Continued on Next Page.

Table 4: Distance function estimation: U.S.

Dependent variable: $\ln(1/u_1)$	Coef.	Std. Err.	z	$P > z$	95% Conf. Interval
<i>Alabama</i>	0.106	0.010	10.620	0.000	0.087 0.126
<i>Alaska</i>	-0.004	0.038	-0.110	0.916	-0.079 0.071
<i>Arizona</i>	-0.053	0.023	-2.300	0.022	-0.098 -0.008
<i>Arkansas</i>	0.095	0.010	9.640	0.000	0.075 0.114
<i>California</i>	-0.036	0.010	-3.740	0.000	-0.055 -0.017
<i>Colorado</i>	0.031	0.012	2.450	0.014	0.006 0.055
<i>Connecticut</i>	-0.148	0.024	-6.240	0.000	-0.194 -0.101
<i>Delaware</i>	-0.167	0.034	-4.910	0.000	-0.233 -0.100
<i>Florida</i>	0.000	0.010	0.000	1.000	-0.019 0.019
<i>Georgia</i>	0.009	0.008	1.140	0.255	-0.007 0.025
<i>Idaho</i>	0.182	0.016	11.670	0.000	0.151 0.212
<i>Illinois</i>	0.018	0.008	2.430	0.015	0.004 0.033
<i>Indiana</i>	0.097	0.010	9.960	0.000	0.078 0.117
<i>Iowa</i>	0.031	0.008	3.700	0.000	0.015 0.047
<i>Kansas</i>	0.100	0.009	11.420	0.000	0.083 0.117
<i>Kentucky</i>	0.085	0.009	9.300	0.000	0.067 0.103
<i>Louisiana</i>	0.151	0.009	15.900	0.000	0.132 0.169
<i>Maine</i>	-0.014	0.019	-0.740	0.459	-0.051 0.023
<i>Maryland</i>	0.125	0.016	7.950	0.000	0.094 0.155
<i>Massachusetts</i>	0.030	0.008	3.900	0.000	0.015 0.045
<i>Michigan</i>	0.044	0.011	4.120	0.000	0.023 0.066
<i>Minnesota</i>	-0.027	0.011	-2.540	0.011	-0.048 -0.006
<i>Mississippi</i>	0.057	0.011	5.140	0.000	0.035 0.079
<i>Missouri</i>	0.113	0.008	14.430	0.000	0.097 0.128
<i>Montana</i>	0.082	0.014	5.970	0.000	0.055 0.108
<i>Nebraska</i>	0.057	0.010	5.640	0.000	0.037 0.077
<i>Nevada</i>	-0.024	0.024	-1.040	0.300	-0.070 0.022
<i>New Hampshire</i>	0.053	0.024	2.230	0.026	0.006 0.100
<i>New Jersey</i>	-0.038	0.012	-3.090	0.002	-0.062 -0.014
<i>New Mexico</i>	0.097	0.019	5.140	0.000	0.060 0.134
<i>New York</i>	-0.004	0.011	-0.340	0.731	-0.025 0.018
<i>North Carolina</i>	0.006	0.012	0.510	0.613	-0.017 0.029
<i>North Dakota</i>	0.089	0.012	7.230	0.000	0.065 0.114
<i>Ohio</i>	0.126	0.011	11.490	0.000	0.104 0.147
<i>Oklahoma</i>	0.144	0.009	16.480	0.000	0.127 0.161
<i>Oregon</i>	0.179	0.016	11.090	0.000	0.147 0.211
<i>Pennsylvania</i>	0.048	0.010	4.640	0.000	0.028 0.068
<i>Rhode Island</i>	-0.052	0.058	-0.910	0.364	-0.166 0.061
<i>South Carolina</i>	0.007	0.012	0.560	0.574	-0.017 0.031
<i>South Dakota</i>	0.058	0.017	3.430	0.001	0.025 0.092
<i>Tennessee</i>	0.068	0.009	7.240	0.000	0.050 0.087
<i>Texas</i>	0.080	0.008	10.230	0.000	0.065 0.096
<i>Utah</i>	-0.064	0.030	-2.140	0.032	-0.123 -0.005
<i>Vermont</i>	0.157	0.020	7.780	0.000	0.117 0.197
<i>Virginia</i>	0.114	0.011	10.110	0.000	0.092 0.136
<i>Washington</i>	0.056	0.012	4.640	0.000	0.033 0.080
<i>West Virginia</i>	0.133	0.014	9.600	0.000	0.106 0.160
<i>Wyoming</i>	(omitted)				
<i>Constant</i>	1.279	0.205	6.250	0.000	0.877 1.680

Also in this case the coefficients of inputs and outputs are significant and with correct sign.

Making a comparison with Europe we can say that in the U.S. there is a greater

impact of labor (-0.86 vs -0.77) and the opposite for capital (-0.22 vs -0.47). This means that the labor factor (capital factor) in the U.S. performs more (less) than in Europe, in fact, increasing the latter the negative effect on efficiency is more limited. This is probably due to the lower dimension of capital employed in Europe compared to labor.

If we analyze the spatial dummies, there are two groups of countries placing above and below the baseline country (Wyoming) in terms of efficiency level. This leads us to question on the inefficiencies and responsibilities of countries and banks.

6. Inefficiencies and responsibilities

In this section we report and comment the results of efficiency evaluations obtained for Europe and the U.S..

First, the distance function satisfies all constraints listed in section 4.

Table 5 shows the ranking of efficiency of banks in Europe⁹ and Table 6 in the U.S.. The efficiency score for Europe and the U.S. has been calculated as average over banks.

Table 5: Efficiency: Europe (Time average data)

Country	Efficiency	Country	Efficiency
<i>Great Britain</i>	0.8136	<i>Denmark</i>	0.8116
<i>Germany</i>	0.8133	<i>Sweden</i>	0.8116
<i>Austria</i>	0.8132	<i>Portugal</i>	0.8113
<i>France</i>	0.8132	<i>Ireland</i>	0.8112
<i>Belgium</i>	0.8129	<i>Luxembourg</i>	0.8108
<i>Italy</i>	0.8128	<i>Switzerland</i>	0.8101
<i>Greece</i>	0.8120	<i>Spain</i>	0.8078
<i>Netherlands</i>	0.8117	<i>Europe</i>	0.8120

Table 6: Efficiency: The U.S. (Time average data)

Country	Efficiency	Country	Efficiency	Country	Efficiency
<i>Wyoming</i>	0.8507	<i>Arkansas</i>	0.8456	<i>Virginia</i>	0.8445
<i>Nevada</i>	0.8496	<i>Georgia</i>	0.8453	<i>New Jersey</i>	0.8445
<i>Connecticut</i>	0.8480	<i>Alaska</i>	0.8453	<i>North Carolina</i>	0.8443
<i>New Mexico</i>	0.8476	<i>Missouri</i>	0.8452	<i>Wisconsin</i>	0.8443

Continued on Next Page

⁹We omit in table 5 Finland, Norway and Turkey given the limited available number of banks for these countries.

Table 6: Efficiency: The U.S. (Time average data)

Country	Efficiency	Country	Efficiency	Country	Efficiency
<i>Louisiana</i>	0.8470	<i>Iowa</i>	0.8452	<i>Michigan</i>	0.8443
<i>Montana</i>	0.8466	<i>Oregon</i>	0.8451	<i>Alabama</i>	0.8439
<i>North Dakota</i>	0.8465	<i>Texas</i>	0.8451	<i>Florida</i>	0.8438
<i>California</i>	0.8465	<i>Maryland</i>	0.8450	<i>Minnesota</i>	0.8437
<i>Mississippi</i>	0.8464	<i>Illinois</i>	0.8450	<i>Vermont</i>	0.8436
<i>Tennessee</i>	0.8461	<i>Oklahoma</i>	0.8450	<i>Colorado</i>	0.8436
<i>Ohio</i>	0.8461	<i>Pennsylvania</i>	0.8450	<i>Maine</i>	0.8435
<i>Idaho</i>	0.8460	<i>Nebraska</i>	0.8450	<i>New York</i>	0.8433
<i>South Carolina</i>	0.8459	<i>Kansas</i>	0.8449	<i>Washington</i>	0.8430
<i>Massachusetts</i>	0.8459	<i>Kentucky</i>	0.8449	<i>Delaware</i>	0.8409
<i>South Dakota</i>	0.8458	<i>West Virginia</i>	0.8448	<i>Utah</i>	0.8374
<i>New Hampshire</i>	0.8457	<i>Indiana</i>	0.8447		
<i>Rhode Island</i>	0.8456	<i>Arizona</i>	0.8446	U.S.	0.8451

We note that the average value of efficiency is high both in Europe (0.812) and even more in the U.S. (0.845) with a slight difference of about 3%. However, such a result may be due once again - as seen for the efficiency performance of Ireland in Europe - to the fact that the distance function does not consider the default risk. Actually, the *NPLs* price may well be different for the banks of these two countries even if the efficiency is similar. To verify this it suffices to think that the efficiency is defined by the distance function while the *NPLs* price (normalized) by its first derivative. Pastor and Serrano (2006) arrived to the same conclusion although with a non parametric approach, which shows that our result is not peculiar to the methodology here adopted. With the aim to consider a measure of the default risk we calculate the shadow prices of the *NPLs*. In Tables 7¹⁰ and 8 are shown shadow prices of the *NPLs* in absolute value.

Table 7: Shadow price of NPL: Europe (Time average data)

Country	P_{NPL}^0	Country	P_{NPL}^0
<i>Sweden</i>	0.3281	<i>Denmark</i>	0.1394
<i>Netherlands</i>	0.2684	<i>Austria</i>	0.1167
<i>Belgium</i>	0.2392	<i>Germany</i>	0.1167
<i>Switzerland</i>	0.2138	<i>Italy</i>	0.1084
<i>Luxembourg</i>	0.2042	<i>Spain</i>	0.1057
<i>France</i>	0.1822	<i>Greece</i>	0.0821
<i>Great Britain</i>	0.1770	<i>Portugal</i>	0.0371
<i>Ireland</i>	0.1551	Europe	0.1580

¹⁰We omit in table 7 Finland, Norway and Turkey given the limited available number of banks for these countries.

Table 8: Shadow price of NPL: The U.S. (Time average data)

Country	P_{NPL}^o	Country	P_{NPL}^o	Country	P_{NPL}^o
New Hampshire	0.3987	Mississippi	0.2395	Michigan	0.2060
Vermont	0.3852	Arizona	0.2367	Delaware	0.2055
Rhode Island	0.3795	California	0.2327	Tennessee	0.2051
Maine	0.3722	Louisiana	0.2316	North Carolina	0.2047
New Jersey	0.3511	South Carolina	0.2302	Idaho	0.2041
Pennsylvania	0.3300	Washington	0.2290	Arkansas	0.1979
Alaska	0.2988	Iowa	0.2273	Georgia	0.1969
Connecticut	0.2986	Ohio	0.2261	Nebraska	0.1963
Wyoming	0.2691	Illinois	0.2248	Nevada	0.1913
Virginia	0.2688	Kentucky	0.2232	Alabama	0.1877
Indiana	0.2679	West Virginia	0.2228	Oklahoma	0.1872
Maryland	0.2639	Montana	0.2221	Kansas	0.1868
Oregon	0.2586	Minnesota	0.2205	New Mexico	0.1865
New York	0.2583	Missouri	0.2199	North Dakota	0.1740
Massachusetts	0.2575	Florida	0.2153	Utah	0.1672
Wisconsin	0.2513	Colorado	0.2143		
South Dakota	0.2406	Texas	0.2138	U.S.	0.2272

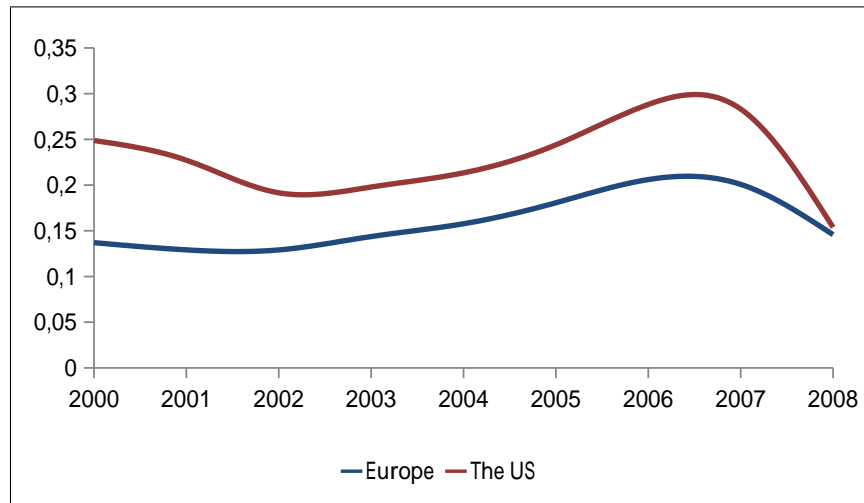


Figure 2: Shadow price of *NPLs* series of Europe and the U.S. (years 2000-2008)

As can be seen from Tables 7 and 8 even if the banks of our sample are much efficient and even more those one of the U.S., this result may be reasonably due to a risky management of the credit activity. In fact, the average shadow price of *NPL* is quantitatively relevant in both areas but more costly in the U.S.. More specifically, on average, the cost of the debt collection amounts to 22% in the

U.S. and 16% in Europe.

The graph in Figure 2 shows how in both cases the price of *NPLs* has greatly risen since 2002 with a peak between years 2006 and 2007 and has fallen during 2008 for the regulatory actions of the governments as a consequence of the crisis. In 2008 the two *NPLs* prices become almost equal.

But who is the responsible between countries and banks?

To answer this question we move from the consideration that if the bank risk is controlled across countries by each single bank, then the banking system would be reliable. On the other hand, if the risk doesn't vary across banks per each single country, then the regulation imposed by countries is effective. If the autonomy of the banks to manage with the risk across countries is high, then the responsibility is more referable to banks. This problem can be analyzed in terms of between-variances, applied to the variable representing the risk, evaluated across countries ($\sigma_{B_{countries}}^2$) and across banks ($\sigma_{B_{banks}}^2$): the former represents the capacity of the banks to control the risk and the latter the capacity of countries to set appropriate regulations capable to control the risk. We normalize the former to the latter to make a comparison. We consider two variables of interest, *NPLs/L* and *NPLs* price, and conclude that the higher is the $\sigma_{B_{countries}}^2 / \sigma_{B_{banks}}^2$ ratio the less the attention devoted by countries to the control the risk of the banking system.

We set up an analysis of variance of both the ratio *NPLs/L* and *NPLs* shadow prices by decomposing the total variance for *banks* and *countries*, and found that in both cases the between variance is the largest one.

Table 9: Analysis of variance of *NPLs/L*
 $\left(\sigma_{B_{countries}(NPLs/L)}^2 / \sigma_{B_{banks}(NPLs/L)}^2 \right)$

	Europe	U.S.
2000	2.358	0.036
2001	2.062	0.033
2002	2.232	0.024
2003	2.384	0.010
2004	2.367	0.002
2005	2.299	0.001
2006	2.302	0.001
2007	1.972	0.002
2008	1.950	0.014

Table 10: Analysis of variance of the shadow price of the NPLs

$$\left(\sigma_{B_{countries}(P_{NPLs})}^2 / \sigma_{B_{banks}(P_{NPLs})}^2 \right)$$

	Europe	U.S.
2000	0.431	5.516
2001	0.096	3.180
2002	0.098	1.264
2003	0.202	1.704
2004	0.199	2.049
2005	0.221	3.020
2006	0.286	2.897
2007	0.221	2.088
2008	0.362	0.383

Tables 9 and 10 show that European countries are careful with the *NPLs* management ($\sigma_{B_{countries}}^2(NPLs/L) > \sigma_{B_{banks}}^2(NPLs/L)$), while the quality of *NPLs* - *i.e.* their price - is defined by the banks ($\sigma_{B_{countries}}^2(P_{NPLs}) < \sigma_{B_{banks}}^2(P_{NPLs})$).

Surprisingly we obtain for the two variables two unequivocally opposite evidences attesting the U.S. system, compared to Europe, as more vigilant on banks default when considering *NPLs/L* and the opposite for the *NPLs* price. The explanation of such apparently different results is that the definition of *NPLs* in the U.S. is not so prudent as in Europe in that in the former case the *NPLs* refer only to the loans declared officially non reimbursable while in the latter one is much more cautious including also the loans declared protested against. Actually the main difference between the two regulatory systems is that, under the US the *generally accepted accounting principles* (GAAP), the *statement financial accounting standard* (SFAS) n.5 defines a very broad and vague criterion to detect the *NPLs*, based on the “probable” and “reasonably estimated” loss. As a consequence the loss provision becomes a strategic variable for banks which may increase it in case of bad evaluation from the markets to show a greater credibility or, on the contrary, may enhance it in the opposite case in order to improve profits by reducing the tax base. This of course artificially lowers or raises the variance across countries of *NPLs/L* of the banks in the U.S. and Europe respectively ¹¹. Instead, according to the Basel agreements II and III, in Europe there is a lower bound of 1.25% of the “risk weighted asset” for the loss provision and

¹¹ Note that such a result does emerge notwithstanding we considered the different definitions of the *NPLs*, in the two countries under exam, after having normalized between the variances.

an upper bound of 50% of the “regulatory capital requirements”¹².

7. Final Remarks and Policy implications

In the analysis developed we discuss about credit market, country’s policy actions and efficiency of the banking system.

With regard to the credit market our analysis identifies an increasing *NPLs* price in the considered period as showed in Figure 2 and underlines that in the usual risk analysis is difficult to take properly into account this trend, being the *NPLs* price not normally observable.

Moreover, comparing the *NPLs* price with the interest rate of loans, we reckon that banks measure incorrectly the real risk and the cost to recover the *NPLs* by fixing an interest rate that does not contain adequately the effective *NPLs* price. Given such an excessive cost, it would be appropriate to monitor the lending banks policy with apposite regulations which take into account the *NPLs* price as a margin to be stored in case of loss.

A second point is that there is the necessity to homogenize the definition of *NPLs* in order to avoid that the ratio *NPLs*/*L* is systemically and artificially different between countries, like in the case considered here where such ratio is sensibly lower in the US compared to Europe. On the contrary, our analysis shows that the *NPLs* price is always higher in the U.S. with respect to Europe.

A significant result of our research the importance of countries in explaining the recent financial crisis. From Table 10 in the U.S. the $\sigma_{B_{countries}}^2(P_{NPLs})/\sigma_{B_{banks}}^2(P_{NPLs})$ ratio is very high from 2000 to 2007 showing a great responsibility of the countries in not having preserved a homogeneous risk management of banks (low $\sigma_{B_{banks}}^2(P_{NPLs})$). This is confirmed by the low *NPLs*/*L* ratio obtained in 2008 when the U.S. government intervened by introducing market-wide support measures and assisting failing financial institutions. In light of these facts, legislative measures for monitoring the banks would be important to avoid future crisis. In Europe, instead, this supervision was already effective as showed by the low $\sigma_{B_{countries}}^2(P_{NPLs})/\sigma_{B_{banks}}^2(P_{NPLs})$ ratio. Therefore, in order to improve the quality of loans in Europe the direction is to look for some improvements in terms of efficiency. In effect, in such a respect we note that in Europe the risk strategies, concerning the ratio between non performing loans and loans, are very different among banks (high $\sigma_{B_{countries}}^2(NPLs/L)/\sigma_{B_{banks}}^2(NPLs/L)$), which

¹²Moreover, still in the definition of the “risk weighted asset” the weights are more compelling in Europe than in the US.

is likely to be referred to different levels of efficiency in the loans management. Actually, Tables 5 and 6 show a lower efficiency in Europe than in the U.S.. A possible explanation of this fact is that European banks try to bypass the stricter rules on the *NPLs* registration by improving profits with a reduction in the regulatory capital¹³. Hence, an intriguing question, possibly of future research, should be to understand how much part of the banks efficiency is due to the correct proportion between the undervalued *NPLs/L* and the regulatory capital, or how much of inefficiency is due to the correct evaluation of *NPLs/L* in contrast to a low regulatory capital. A way to solve this problem would be to penalize risky banks by asking them to pay as a penalty the *NPLs* price. This would be a counterincentive to the expansion of *NPLs* as a strategy to gather more funds irrespective of the risk.

8. Conclusion

The analysis conducted in this paper showed that the recent bank crisis could be anticipated if appropriate indicators would have been used. We propose here the *NPLs* price which is not observable. Our econometric methodology based on the Fourier expansion validates significantly the theoretical set up adopted. Actually we found that the market interest rates do not adequately account for the risk of loans loss. Further, Europe and the U.S. have different peculiarities concerning the inefficiencies of the bank system and the responsibilities of the two countries. We found more countries' responsibility in terms of low regulations for the U.S. and a slightly more inefficiency for Europe. A proposal to monitor both aspects is to penalize risky banks by asking to pay as a penalty the *NPLs* price.

Acknowledgments

The authors wish to thank Daniela Palatta and Ronald Gallant for helpful comments and suggestions. Financial support is from University of Rome La Sapienza and MIUR.

¹³Slovik (2012) finds the same result on the base of an analysis of the ratio between the risk-weighted assets to total asset.

Bibliography

- Aigner, D. J., Chu, S. F., 1968. On estimating the industry production function. *American Economic Review* 58, 826–839.
- Berger, A. N., 1993. 'distribution free' estimates of efficiency of the u.s. banking industry and tests of the standard distributional assumptions. *Finance and Economics Discussion Series* 188, Board of Governors of the Federal Reserve System (U.S.).
- Berger, A. N., DeYoung, R., 1997. Problem loans and cost efficiency in commercial banks. Tech. rep.
- Berger, A. N., Leusner, J. H., Mingo, J. J., 1997. The efficiency of bank branches. *Journal of Monetary Economics* 40 (1), 141 – 162.
- Cuesta, R. A., Orea, L., 2002. Mergers and technical efficiency in spanish savings banks: A stochastic distance function approach. *Journal of Banking & Finance* 26 (12), 2231–2247.
- Eastwood, B. J., Gallant, A. R., 1991. Adaptive rules for seminonparametric estimators that achieve asymptotic normality. *Econometric Theory* 7 (03), 307–340.
- Färe, R., 1988. Fundamentals of production theory. *Lecture notes in economics and mathematical systems*. Springer.
- Färe, R., Knox Lovell, C. A., October 1978. Measuring the technical efficiency of production. *Journal of Economic Theory* 19 (1), 150–162.
- Färe, R., Primont, D., 1995. *Multi-output production and duality: theory and applications*. Kluwer Academic Publishers.
- Farrell, M. J., 1957. The measurement of productive efficiency. *Journal Of The Royal Statistical Society Series A General* 120 (3), 253–290.
- Gallant, A. R., February 1981. On the bias in flexible functional forms and an essentially unbiased form : The fourier flexible form. *Journal of Econometrics* 15 (2), 211–245.
- Hughes, J. P., Mester, L. J., 1993. A quality and risk-adjusted cost function for banks: Evidence on the too-big-to-fail doctrine. *Journal of Productivity Analysis* 4, 293–315.
- Jacobsen, S. E., June 1972. On shephard's duality theorem. *Journal of Economic Theory* 4 (3), 458–464.
- Lovell, K., Richardson, S., Travers, P., Wood, L., 1994. Resources and functions: A new view of inequality in australia. *School of Economics Working Papers 1990-07*, University of Adelaide, School of Economics.
- Maggi, B., Guida, M., 2011. Modelling non-performing loans probability in the commercial banking system: efficiency and effectiveness related to credit risk in italy. *Empirical Economics* 41, 269–291.
- Pastor, J., 2002. Credit risk and efficiency in the european banking system: A three-stage analysis. *Applied Financial Economics* 12 (12), 895–911.
- Pastor, J., Serrano, L., 2005. Efficiency, endogenous and exogenous credit risk in the banking systems of the euro area. *Applied Financial Economics* 15 (9), 631–649.
- Pastor, J., Serrano, L., 2006. The effect of specialisation on banks' efficiency: An international comparison. *International Review of Applied Economics* 20 (1), 125–149.
- Shephard, R., 1970. *Theory of cost and production functions*. Princeton studies in mathematical economics. Princeton University Press.
- Slovik, P., 2012. Systematically important banks and capital regulations challenges. *Department Working Papers*, OECD Publishing 916.