Inflation and CO2 emissions

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Abstract

This work aims to forecast the inflation extending the application of the Dynamic Moving Average (DMA) model to account for CO2 emission. The DMA, whose basis in this case is the generalized Phillips curve equation, is a TVP-model which allows parameters, explanatory variables and the whole model itself to change through the entire time considered. Its application for the forecast of the quarterly US inflation rate has been explored with the addition, in the set of explanatory variables, of a climate change indicator: the level of CO2 emissions of the country. Our results show point toward the goodness of the DMA model to forecast inflation when this new variable is added, showing the relevance of the study of the impact of climate change for the macroeconomics.

1 Introduction

The last few years have been extremely challenging for the role of the statistician, given the various extraordinary events that monopolised the news in these periods. One of the problems we are facing at present time is the rising to alarming levels of the inflation rates of all major currencies of the world.

Inflation is the rate of increase in prices over a given period of time. It is typically a broad measure, such as the overall increase in prices or the increase in the cost of living in a country[1]. Inflation affects all aspects of the economy,

[1] IMF definition

from consumer spending, business investment and employment rates to interest rates, tax and government policies. It is important for policy makers, like central banks, because one of their main purposes is to keep inflation rates within a prefixed interval (usually around 2%) in order to maintain economy stable. It is important for all investors in the financial market because it erodes the real value of investor capital and investments. It could also make future company profits less valuable today and lead to higher interest rates; this too is a big concern for investors since it is known that in times of high inflation there will be rising interest rates. Inflation is also important for employees and households, because rising inflation rates mean a decline in their purchase power and the value of their savings.

An accurate forecast of future inflation rates is vital for all players in the economy. Economists approached this problem widely during decades, from the first Phillips curve[2] to its generalization, applying various econometric models. The Phillip's curve graphically represents the economic relationship between the rate of change of unemployment and the rate of change of salary wages. This equation, in a more generalized form, has been the base model for many studies, Stock and M. W. Watson (1999), Stock and M. W. Watson (2006), Stock and M. W. Watson (2008), Stock and M. Watson (2009). Their models employed several economic variables picked from the major economic theories. In this work a part of their selected variables has been used in the same way, as the generalized Phillips curve is the base of our model. Concerning the econometric side, we recall Hoogerheide et al. (2010) work, in which it was found that a time-varying parameters model, on economic previsions in general, performs much better than a static one. Primiceri (2005),Chan et al. (2012) Cogley et al. (2010) investigate VAR models with dynamic coefficients and stochastic volatility in inflation forecast exercises. Also Chan (2013) attempt of a moving average model with stochastic volatility and Groen et al. (2013) attempt of a Bayesian moving average that allows for structural breaks in the error variance result in a more accurate prediction output. Thus, there are several empirical results

[2]Phillips (1958)

showing that inflation's parameters and relations with other economic variables has always changed during time. Koop et al. (2012) realized the potential utility of the Dynamic Moving Average model proposed by Raftery et al. (2010) in an engineering application context, and implemented it in a forecasting exercise of inflation rates in the US. This model involves a model updating equation of the generalized Phillips curve using posterior inclusion probabilities obtained using a recursive estimation algorithm. Therefore, it follows the current theories of parameters-changing model for the inflation rates.

This work extends the work of Koop et al. (2012) with the addition of a variable related to climate change, that is the total emissions of CO2 gas in the country. The aim is to discover if there is a relation between the two.

Global warming and climate change have several ecological impacts, such as floods, extreme storms, sea-level rise, and droughts. CO2 emissions contribute to a country's pollution levels. Carbon dioxide (CO2) emissions are greenhouse gases resulting from the burning of fossil fuels. A greenhouse gas (GHG) is a gas that absorbs and emits thermal radiation, creating a "greenhouse effect." While carbon dioxide is important in keeping the Earth a habitable temperature, the excessive CO2 emissions caused by the increasing consumption of fossil fuels disrupt Earth's carbon cycle and accelerate global warming. Between 2010 and 2019, total global CO2 emissions have increased from 33.1 gigatons to 38 gigatons and are projected to increase in the coming years. Unfortunately, CO2 emissions are increasing worldwide, and the nations that are emitting the highest amounts are not doing enough to reduce emissions. Several works in economic and econometric shed light to this very important issue to share awareness on this impelling world-wide problem, focusing especially on the impact on transmission of monetary policies and prices. We recall Faccia et al. (2021), who highlights three main channels in which climate change effects impact price stability:

- The more frequent cadence of disastrous events (wildfires, floods, storms, etc) affect the price of primary foods, increasing their price for both the producer and the consumer;

- The transition to a zero carbon production increases the cost of carbon, which transmits into higher bills for citizens and higher cost for changes in production for industrial firms;

- These apocalyptic events have an enormous toll in terms of human lives. This unfortunately also translates into a reduction of labor force and territories to be used for production.

Also Batten et al. (2020) divided between direct (physical and transition risks that can affect the macro-economy and the prospects for inflation) and indirect (impact on households and firms' expectations about future economic outcomes) effects of climate change into monetary policies. Therefore, there are evidences of the extend and importance of the well-being of our planet, not only based on scientific and human side arguments, but there is also an undeniable connection to the global and local economy that should be considered in every econometric analysis.

In our empirical work, we find evidence of the significance of CO2 emissions into the DMA model, especially in the 80's and 90's for the longer horizon forecast. From empirical analysis, the role of a pollution indicator brings consistency into the forecast, and proves to be more significant than other macroeconomic variables included by literature. In addition, we found additional proofs on the reliability of the DMA model for the econometric analysis of the Phillips curve, corroborating the previous findings of Koop et al. (2012): the inflation rate seems to follow a process with time-varying parameters and thus non-stationary.

The rest of the paper is structed as follows. Section 2 gives an exhausting explanation of how the DMA model is structured, focusing first on the basis from whom it was conceived and implemented, and secondly explaining its functioning. Section 3 presents an empirical analysis of the results of the forecast exercise, focusing on descriptive analysis and and empirical interpretation of the results. Section 4 concludes the discussion.

2 The model

2.1 Generalized Phillips Curve Model

The foundation of the model used in this paper is the *generalized Phillips curve*[3]

$$y_t = \phi + x'_{t-1}\beta + \epsilon_t \tag{1}$$

where y_t is the inflation, transformed as $100\log(\frac{P_t}{P_t - 1})$, where P_t is one of the two variables chosen to describe inflation, and x_t is a vector of predictors. When forecasting $h > 1$ times ahead, the equation becomes

$$y_t = \phi + x'_{t-h}\beta + \epsilon_t \tag{2}$$

with $y_t = (100/h)\log(P_t/P_{t-h})$. Following Koop et al. 2012, two measures of inflation are employed in this work: the Gross Domestic Product (GDP) deflator and Personal Consumption Expenditure (PCE) deflator[4]. Concerning the explanatory variables, following the literature above, a list of macroeconomic indicators, measures of real activities, a survey of inflation expectations and lagged inflation were used inside matrix of regressors. As pointed out by Stock and M. W. Watson (1999), cite: "The only variables that consistently improve upon Phillips curve forecasts are measures of aggregate activity, and the best of these is a new index of 168 indicators of economic activity. These alternative forecasts, when combined with Phillips curve forecasts, produce forecasting gains that are both statistically and economically significant." All variables included in this model are contained in the above-mentioned index.

2.2 Explanatory Variables for Inflation

The selection of the variables included in the regression follows Koop et al. (2012) plus the new addition, which is the base of the extended work pursued, of an "environmental indicator", that is the emissions of CO2 gas for the United

[3] Stock and M. W. Watson (1999)

[4] They will be called `INFL_GDP` and `INFL_PCE` in mnemonic and code

Mnemonic	Description
UNEMP	Unemployment Rate
CONS	Real Personal Consumption Expenditures: Total
INV	Real Gross Private Domestic Investment: Residential
GDP	Real Gross Domestic Product
HSTARTS	Housing Starts
EMPLOY	NonFarm Payroll Employment
MONEY	M1 Money Stock
PMI	ISM Manufacturing: PMI Composite Index
TBILL	3-Month Treasury Bill: Secondary Market Rate
SPREAD	Spread 10 years T-Bond yield/3 month T-Bill
INFEXP	University of Michigan Inflation Expectations
co2EMIS	CO2 Total Emissions (Mt CO2/year)

Table 1:

States. Following, a brief explanation of the variables and their economic connection with the dependent variable y_t.

Unemployment Rate Unemployment is the situation that economists refer to when the number of jobless people who are willing to work exceeds the supply of jobs in the workforce. The unemployment rate is the core of the very first Phillips curve. This single-equation economic model, named after its inventor Alban William Phillips, shows the inverse trade-off between rates of inflation and rates of unemployment. If inflation is high, unemployment will be low; if unemployment is high, unemployment will be low. This happens because when unemployment is low, the demand for workers exceeds the number available. When unemployment rises, on the other hand, the availability of individuals looking for work exceeds demand. That's because not many employers are hiring even if more people want to get to work. Even though in the long run it has been proven that they are unrelated, in the short run it remains true. The forecast exercises in this work have horizons $h = 1$ (one quarter), $h = 4$ (one year) and $h = 8$ (two years), so our timeline falls into the short run.

Gross Domestic Product Empirical studies conducted for several countries (both industrial and developed) have shown ambivalent results: some found a negative relation that could not be extended through all scenarios, periods of time and countries; other show a possible positive relation between inflation and economic growth. Granger causality has also been explored in both ways. Koulakiotis et al. (2012) did an all-time review of all these various positions for European countries, findings a bilateral connection of cause-effect.

Real Personal Consumption Expenditures Real personal consumption expenditure is the primary measure of consumers' spending on goods and services in a given period of time.It gives us a measure of how much people are spending, and can also give an inside of the money power held by the people. In 2012, the PCE Price Index became the primary inflation index used by the U.S. Federal Reserve when making monetary policy decisions. The Personal Consumption Expenditures Price Index is a measure of the prices that people living in the United States, or those buying on their behalf, pay for goods and services. The change in the PCE price index is known for capturing inflation (or deflation) across a wide range of consumer expenses and reflecting changes in consumer behavior. Since it is one of the most used deflators for price index as a measure of inflation, it is unmistakably related to it.

Real Gross Private Domestic Investment Gross private domestic investment is a useful component for the formation of the GDP. It measures the amount of money that domestic business invest in their own country. It is used to give an overall number of the business activities in the country. Since it is part of the GDP formation, it may be useful to see if this variable in particular can give us more insight on the topic.

Housing Starts This variable measures the number of new residential houses projects started in the given period. It is used always as an economic indicator of the economic well-being of a nation. It includes building permits, construction starts and completation of house projects.

NonFarm Payroll Employment Nonfarm payrolls defines the measure of the number of workers in the U.S. excluding farm workers and workers in a handful of other job classifications. This is a survey on private and government entities throughout the U.S. about their paysheets. In addition to farm workers, nonfarm payrolls data also excludes some government workers, private households, proprietors, and non-profit employees.

ISM Manufacturing: PMI Composite Index The purchasing managers' index is another indicator of economic activity of the country, issued by the Institute for Supply Management. It is a survey of the entity of the manufacturing activities by more than 300 manufacturing firms in 20 different sectors. It's considered one of the main indicator of the ongoing economic situation in the country. A value below 50 might indicate signs of economic recession, especially if the tendency lasts for many months.

3-Month Treasury Bill: Secondary Market Rate The 3-Month Treasury bill is a short-term U.S. government security with a constant maturity period of 3 months. This index focuses on the buying and selling in the secondary market. The primary market is the place where stocks and bonds are issued for the first time, while the secondary market is where the most exchange of all financial items happens. It is known that interest rates are the primary tool of central banks when trying to fight unwanted inflation rate changes.

Inflation Expectations Inflation expectations are the rate at which people, consumers, businesses, and investors expect prices to rise in the future. They matter because actual inflation depends, in part, on what we expect it to be. If everyone expects prices to rise, say, 3 percent over the next year, businesses will want to raise prices by (at least) 3 percent, and workers and their unions will want similar-sized raises. All else equal, if inflation expectations rise by one percentage point, actual inflation will tend to rise by one percentage point as well. How people expect prices to develop in the future influences how they spend, borrow and invest money today. This in turn affects the economy and is therefore important for our monetary policy. If we understand people's inflation expectations better, we can make more informed predictions on the movement and expectations for inflation.

CO2 Emissions Carbon dioxide (CO2) is a colourless, odourless and non-poisonous gas created by combustion of carbon and in the respiration of living organisms and is considered a greenhouse gas. Greenhouse gases constitute a group of gases contributing to global warming and climate change. CO2 was included in this class in 1977 during the Kyoto Protocol, a major environmental agreement adopted by many of the parties to the United Nations Framework Convention on Climate Change (UNFCCC). Emissions means the release of greenhouse gases and/or their precursors into the atmosphere over a specified area and period of time. Carbon dioxide emissions or CO2 emissions are emissions stemming from the burning of fossil fuels and the manufacture of cement; they include carbon dioxide produced during consumption of solid, liquid, and gas fuels as well as gas flaring.

2.3 Kalman Filtering

The Kalman filter is one of the most important and common estimation algorithms of state space models. It produces estimates of hidden variables based on inaccurate and uncertain measurements. Also, the Kalman Filter provides a prediction of the future system state based on past estimations. This state estimation technique was first proposed by Kalman (1960) and ever since it has been object of many extensions and applications in various fields, such as guidance, navigation, and control of vehicles; time series analysis and forecasting; robotic motion planning and Artificial Intelligence. A Kalman filter takes in information which is known to have some error, uncertainty, or noise. The goal of the filter is to take in this imperfect information, sort out the useful parts of interest, and to reduce the uncertainty or noise. The state space model is composed by a transition equation, which describes the state at time t as a linear combination of its past values and a noise term, and a measurement equation, which describes the relation between the observer and the state. In this work, the equations are in the form:

$$y_t = z_t^T \theta_t + \epsilon_t \tag{3a}$$

$$\theta_t = \theta_{t-1} + \eta_t \tag{3b}$$

To be specific, (3a) is the equivalent of (2) with the application of the m x 1 vector of coefficients (states) θ_t; y_t is always the inflation and $z_t = [1, x_{t-h}]$ is the 1 x m vector of predictors in which stationary transformations of the explanatory variable are carried out[5]. Equation (3b) is essential for the application of the Kalman Filter. $\epsilon_t \sim N(0, H_t)$ and $\eta_t \sim N(0, Q_t)$. ϵ_t and η_t are assumed to be mutually independent at all leads and lags.

The algorithm works in two phases. The first is known as the prediction step (eq.(3b)). An initial prior value is given, and run into the measurement equation to produce a measurement estimate which is in fact the prediction for the state of the system at the next time step. Then we incorporate the information derived from our measurement system in the first (eq.(3a)) equation, taking into account the noise covariance matrix that propagates into the system. After, the new information given is filtered of the noise derived from the measurement, taking into account also the noise from the estimate to produce a clean data that is incorporated in the state space observation equation model to produce a final first estimate of the variable. [Update Step]. This will become the base for the second iteration of the algorithm and so on. Thus, the Kalman algorithm is a recursive algorithm, meaning that it always repeats itself, adding, iteration by iteration, more accurate estimates of the phenomenon.

[5] See Section 3.1 for specifics.

2.4 Time-Varying Parameter Models

TVP models are a widely-used class of models used in time series analysis[6]. Their feature of allowing multiple changes in time is of great interest in various applications. A variety of interesting econometric applications of TVP models appeared in recent years; for example, Primiceri (2005) used time-varying structural VAR models in a monetary policy application. Terui et al. (2002) generalize the least squares model weights by reformulating the linear regression model as a state-space specification where the weights are assumed to follow a random walk process. Hoogerheide et al. (2010) analysed forecast accuracy of several time-varying Bayesian models, showing how they outperform more static models. A huge advantage of TVP models is their flexibility in capturing gradual changes. Also, they are dynamic mixture models, so it is possible to use an estimation algorithm for posterior probabilities. Another well-known model in this class is the Bayesian Model Average (BMA)Fragoso et al. (2018). In fact, DMA is the dynamic equivalent of BMA, where the correct model and its parameters are fixed but unknown. We recall, for the sake of completeness, that Bayesian Model Average is an extension of the usual Bayesian inference methods, in which parameters uncertainty is modelled through prior distribution, and uses Bayes theorem to model posterior probabilities that allow for direct model selection, combined estimation and prediction.

2.5 Dynamic Moving Average

2.5.1 Uni-Modal Case

The Dynamic Moving Average model was first set up by Raftery et al. (2010) for an engineering application. Another model presented in the paper is DMS (Dynamic Moving Selection), which slightly differs from the first as it does no averaging, but selects the best model. Specifications for it will be provided when different from the discussion run for DMA. Let y_t[7] and $(z_{tj} : j = 1, ..., m)$ with

[6] Chan et al. (2012)

[7] For specifications on variables, see Section 3.1

$z_{t1} = 1$ corresponding to the regression intercept. The *observation equation* is :

$$y_t = z_t^T \theta_t + \epsilon_t \tag{4}$$

where superscript T denotes matrix transpose, θ_t vector of parameters for the regression and innovations $\epsilon_t \sim N(0, H_t)$. The regression parameters follow the *state equation*:

$$\theta_t = \theta_{t-1} + \eta_t \tag{5}$$

where state innovations $\eta_t \sim N(0, Q_t)$. Inference is done recursively using Kalman algorithm updating technique. Suppose that $\theta_{t-1}|Y^{t-1} \sim N(\hat{\theta}_{t-1}, \Sigma_{t-1|t-1})$ where $Y^{t-1} = \{y_1, ..., y_{t-1}\}$. Then

$$\theta_t|Y^{t-1} \sim N(\hat{\theta}_{t-1}, \Sigma_{t|t-1}) \tag{6}$$

where

$$\Sigma_{t|t-1} = \Sigma_{t-1|t-1} + Q_t \tag{7}$$

Groen et al. (2013) used a similar approach, adding in (4) a dummy variable with values 1 if the i_th predictor is included in the model and 0 otherwise but for all points in time. DMA evolves from this as it allows the set of predictors to change over time. Instead of a static dummy variable, we will calculate a time-varying weight as the probability for each regressor to be included at time t. Details on this feature will be provided shortly after. Focusing on Q_t, Raftery et al. (2010) offer an innovative simplification by letting the specification in (7) be done using a form of *forgetting*, and replacing it with

$$\Sigma_{t|t-1} = \lambda^{-1}\Sigma_{t-1|t-1} \tag{8}$$

where λ is the *forgetting factor* and is usually slightly below 1. The name "forgetting factor" is suggested by the fact that this specification implies that observations j periods in the past have weight λ^j. Consequently $Q_t = a\Sigma_{t-1|t-1}$, with $a = (\lambda^{-1} - 1)$. This definition falls in the structure of state-space model. This means that we don't have to estimate Q_t. Instead, we are required only to run the Kalman filter and estimate H_t. Last, we have the *updating equation*

$$\theta_t|Y^t \sim N(\hat{\theta}_t, \Sigma_{t|t}) \tag{9}$$

Specifically, in (9)

$$\hat{\theta}_{t|t} = \hat{\theta}_{t|t-1} + \Sigma_{t|t-1} z_t^T (H_t + z_t^T \Sigma_{t|t-1} z_t)^{-1} e_t \tag{10}$$

where $e_t = y_t - z_t^T - \hat{\theta_{t-1}}$ is the one step ahead prediction error and

$$\Sigma_{t|t} = \Sigma_{t|t-1} - \Sigma_{t|t-1} z_t (H_t + z_t^T \Sigma_{t|t-1} z_t)^{-1} z_t^T \Sigma_{t|t-1} \tag{11}$$

Inference process is repeated recursively[8] as new observations are added. Recursive forecasting is done using the predictive distribution

$$y_t | Y^{t-1} \sim N(z_t \hat{\theta}_{t-1}, H_t + z_t^T \Sigma_{t|t-1} z_t) \tag{12}$$

One of the few parameters to be set initially are the initial values $\hat{\theta_0}$ and Σ_0 for the first step, that need to be specified for each model k. We followed again Raftery et al. (2010), who approximates using the data themselves. Following the linear regression assumptions, the coefficient of a regression for an input variable X is approximately less than the standard deviation of the system output divided by the standard deviation of X. So, $\hat{\theta_0}^{(k)} = 0$ for each k, and $\Sigma_0^{(k)} = diag(s_1^{2(k)}, ..., s_{\mu k}^{2(k)})$ and $s_j^{2(k)} = (Var(y_t)/Var(z_{t,j}^{(k)}))$ for $j = 2, ..., m$.

2.5.2 Multi-Modal Case

The idea behind DMA is the possibility of letting different models hold at different time and averaging between models through the use of posterior probabilities. Notation will now be switched to implement discussion for a multi-modal case[9]. The multi-model equation from (3a) and (3b) used in this work is explained by the following:

$$y_t = z_t^{(k)} \theta_t^{(k)} + \epsilon_t^{(k)} \tag{13a}$$

$$\theta_{t+1}^{(k)} = \theta_t^{(k)} + \eta_t^{(k)} \tag{13b}$$

where $\epsilon_t^{(k)} \sim N(0, H_t^{(k)})$ and $\eta_t^{(k)} \sim N(0, Q_t^{(k)})$. Let $L_t \in \{1, 2, ..., K\}$ denote which model applies at each time period, $\Theta_t = (\theta_t^{(1)'}, ..., \theta_t^{(k)'})'$ and

[8] As stated before, Kalman filter is a type of recursive optimization algorithm

[9] Explanation for DMS is equal unless specified.

$y^t = (y_1, ..., y_t)'$. Forecasting at time t using information at time $t-1$ involves calculating $Pr = (L_t = k|y^{t-1})$ for $k = 1, ..., K$ number of all possible model combinations. When we calculate $Pr(L_t = k|y^{t-1})$ for $k = 1, ..., K$, the model produces a forecast given by the average forecasts across the models using these probabilities. DMS, instead, selects the single model with the highest value and uses that to forecast. Details on calculation of $Pr = (L_t = k|y^{t-1})$ will be provided later. A brief discussion on the aforementioned computational problem has been widely approached in Raftery et al. (2010). When using switching linear Gaussian state space models, the structure of (13a) and (13b) implies that the state vector Θ_t is divided into different blocks, one for each model k that are independent from each other. The authors exploited this property to derive a solution for the number of times to run the Kalman filter, that is only K.

For the multi-modal case equations are replaced by:

$$\Theta_{t-1}|L_{t-1} = k, y^{t-1} \sim N(\hat{\theta}_{t-1}^{(k)}, \Sigma_{t-1|t-1}^{(k)}) \tag{14a}$$

$$\Theta_t|L_t = k, y^{t-1} \sim N(\hat{\theta}_{t-1}^{(k)}, \Sigma_{t|t-1}^{(k)}) \tag{14b}$$

$$\Theta_t|L_t = k, y^t \sim N(\hat{\theta}_t^{(k)}, \Sigma_{t|t}^{(k)}) \tag{14c}$$

where again $\hat{\theta}_t^{(k)}$, $\Sigma_{t|t-1}^{(k)}$ and $\Sigma_{t|t}^{(k)}$ are obtained following the same procedure using Kalman filter and the (k) subscript denotes model k. For clarification, the prediction and updating equations, conditional on $L_t = k$, provide information only on $\hat{\theta}_t^{(k)}$ and not the full vector of parameters Θ_t. Thus, (14) are written in terms of the distributions that hold for $\hat{\theta}_t^{(k)}$. Discussion until now was focused on conditional probabilities (conditional on $L_t = k$), but we need a method suitable for unconditional probabilities. A common possibility when using Markov-Switching processes and Bayesian inference, (see Hamilton (1989) and others), is to specifying a transition matrix P. Unfortunately, it would require a very big and long computational work, since P would be (in this case) of very great dimension. Raftery et al. (2010) resolved this problem by introducing a forgetting factor comparable to the λ introduced before, α. Back to (12), the conditional probability assumed was:

$$p(\Theta_{t-1}|y^{t-1}) = \sum_{k=1}^{K} p(\theta_{t-1}^{(k)}|L_{t-1} = k, y^{t-1}) Pr(|L_{t-1} = k|y^{t-1}) \quad (15)$$

where $p(\theta_{t-1}^{(k)}|L_{t-1} = k, y^{t-1})$ is given by (14a); a simplification was operated to $\pi_{t|s,l} = Pr(L_t = l|y^s)$ so that the term on the right side of (15) is $\pi_{t-1|t-1,k}$. The transition matrix P of unrestricted probabilities with elements $p_{k,l}$ would be used in the model as:

$$\pi_{t|t-1,k} = \sum_{l=1}^{K} \pi_{t-1|t-1,l} p_{k,l} \quad (16)$$

but thanks to Raftery et al. (2010) we replaced it with

$$\pi_{t|t-1,k} = \frac{\pi_{t-1|t-1,k}^{\alpha}}{\sum_{l=1}^{K} \pi_{t-1|t-1,l}^{\alpha}} \quad (17)$$

where $0 < \alpha \geq 1$ is set to a fixed value slightly below one and its interpretation is similar to the other forgetting factor λ. This simplification led us to a lighter computational burden (again, no MCMC is required). As a result,the model updating equation is:

$$\pi_{t|t,k} = \frac{\pi_{t|t-1,k} p_k(y_t|y^{t-1})}{\sum_{l=1}^{K} \pi_{t|t-1,l} p_l(y_t|y^{t-1})} \quad (18)$$

where $p_l(y_t|y^{t-1})$ is the predictive density for model l evaluated at y_t. Recursive forecasting in DMA is the result of the average of the predictive results for each model using $\pi_{t|t-1,k}$, meaning that scores are given by

$$E(y_t|y^{t-1}) = \sum_{k=l}^{K} \pi_{t|t-1,k} z_t^{(k)} \hat{\theta}_{t-1}^{(k)} \quad (19)$$

For DMS, instead, it selects the single model with the highest value for $\pi_{t|t-1,k}$ at each point in time and uses it for forecasting. It is important to notice how this multi-model prediction of y_t is a weighted average of the model-specific predictions $\hat{y}_t^{(k)}$, where the weights are equal to *the posterior predictive model probabilities for sample t* $\pi_{t|t-1,k}$. As a result of this, model k will receive more weight at time t if its forecast performance in the recent past was successful. Therefore:

$$\begin{aligned}\pi_{t|t-1,k} &\propto [\pi_{t-1|t-2,k} p_k(y_{t-1}|y^{t-2})]^{\alpha} \\ &= \prod_{i=1}^{t-1} [p_k(y_{t-i}|y^{t-i-1})]^{\alpha^i}\end{aligned} \tag{20}$$

The forgetting factor α helps us with controlling the exponential decay at the rate α^i for observations i periods ago to be the same associated with λ. In this case, following Koop et al. (2012) and Raftery et al. (2010), both parameters are set to be equal to 0.99. The last thing to be set is the estimate of H_t. We substitute it with a consistent estimate using an Exponentially Weighted Moving Average (EWMA) model so that,

$$\hat{H}_t^{(k)} = \sqrt{(1-\kappa) \sum_{j=1}^{t} \kappa^{j-1} (y_j - z_j^{(k)} \hat{\theta}_j^{(k)})^2} \tag{21}$$

Literature in this case follows the common use of EWMA estimators to model time-varying volatilities in finance. Details can be found in Riskmetrics (1996). In this work $\kappa = 0.98$ since literature proposed 0.97 for monthly data and 0.94 for daily data[10]. Estimation of $\hat{H}_t^{(k)}$ can be approximated by a recursive form, so it becomes

$$\hat{H}_{t+1|t}^{(k)} = \kappa \hat{H}_{t|t-1}^{(k)} + (1-\kappa)(y_t - z_t^{(k)} \hat{\theta}_t^{(k)})^2 \tag{22}$$

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3 Empirical Analysis

3.1 Data-set

The data set used in this work contains 13 explanatory variables (see Table 1) and two indicators for inflation : Inflation based on GDP deflator and Inflation based on PCE deflator. The entire data set goes from 1978Q1 to 2021Q4, all data are quarterly data. Variables: `INFL_GDP`, `INFL_PCE`, UNEMP, CONS, INV,

[10]In this work data are quarterly data

GDP, HSTARTS, EMPLOY and MONEY (1) were taken from the "Real-Time Data for Macroeconomics" database of the Philadelphia Federeal Reserve Bank. Variables: PMI, TBILL, SPREAD were taken from the FRED database of the St. Louis Federal Reserve Bank. Variable INFEXP index is periodically calculated by the University of Michigan. Variable CO2EMIS was taken from EDGAR, Emissions Database for Global Atmospheric Research, an organ of the European Union. The entire data-set stretches from 1978Q1 to 2021Q4. All data are quarterly data, except for variable CO2EMIS. In this case, quarterly data were not available. Therefore, quarterly data have been obtained using a cubic spline interpolation, which is common use in econometrics (Ajao et al. (2012)). This is a tricky technique because a regression analysis with data that has been estimated will introduce a systematic source of serial correlation in the regression. This problem has been handled using Newey–West standard errors, which are widely used in statistics to overcome the issue of missing standard assumption of regression analysis, and are often used when dealing with time series.

The forecast work is carried out with 3 horizons: $h = 1$ so the forecast is done for the first quarter of 2022;$h = 4$ so the forecast is for the year 2021; and $h = 8$ so the forecast is of 2 years (2020-2021). All the variables have been transformed to be stationary. If we assume $x_{i,t}$ is the raw variable, the transformation applied $z_{i,t}$ are the following:

1. $z_{i,t} = x_{i,t}$ no transformation applied
2. $z_{i,t} = x_{i,t} - x_{i,t-1}-$ first difference
3. $z_{i,t} = x_{i,t} - x_{i,t-2}-$ second difference
4. $z_{i,t} = log x_{i,t}$ logarithm
5. $z_{i,t} = 100 * log(x_{i,t} - x_{i,t-1})$ first difference of logarithm
6. $z_{i,t} = 100 * log(x_{i,t} - x_{i,t-2})$ second difference of logarithm

<u>Transformation 1</u> (no transformation) is applied to:

- Unemployment
- 3 Month Treasury Bill:Secondary Market Rate
- Spread T-Bond/T-Bill
- Inflation Expectations

Transformation 2 is applied to:

- ISM Manufacturing: PMI Composite Index

Transformation 4 is applied to:

- Housing Starts

Transformation 5 is applied to:

- Real Personal Consumption Expenditure
- Real Gross Private Domestic Investment
- Real Gross Domestic Product
- M1 Money Stock

Concerning the dependent variables of the regressions, `INFL_GDP` and `INFL_PCE` for the forecast, they are transformed as: $y_{t+h} = \frac{100}{h}(log(P_{t+h}) - log(P_t))$[11] with P_t being the raw series of data and h is the forecast horizon. Recall that inside the regressor matrix z_t there are also the lagged values of each of the inflation variables; in this case transformation 5 and 6 are applied for lag 1 and 2.

3.2 Forecast Performance

3.2.1 Descriptive Analysis

Here we present some performance tests on the forecasts presented above. The overall goodness of the predicted inflation has been measured with three

[11] It corresponds to transformation 6

different tests. The first, Root Mean Square Forecast Error, (RMSFE), is the expected value of the squared difference between the fitted values implied by the predictive function and the real values of the observed time series. It is an inverse measure of the explanatory power of the predictive function. It is calculated as:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(Predicted_i - Actual_i)^2}{N}} \tag{23}$$

The second is the Mean Absolute Forecast Error, (MAFE), which calculates the sum of absolute errors divided by the sample size :

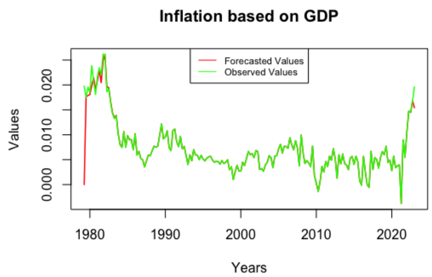
$$\frac{1}{N}\sum_{i=1}^{N}|Actual_i - Predicted_i| \tag{24}$$

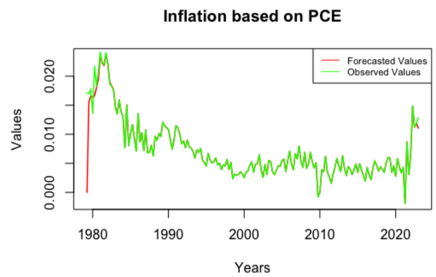
Third, Hit Ratio, which computes the proportion of correctly predicted signs (i.e., in which cases the direction of a change given by forecast agrees with the change in real data). Results are shown in the table below:

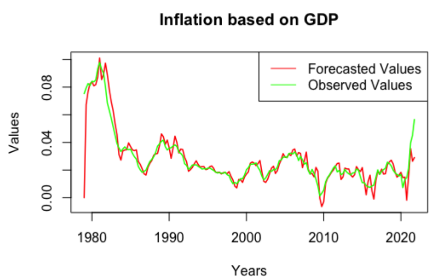
Table 2: Descriptive Statistics

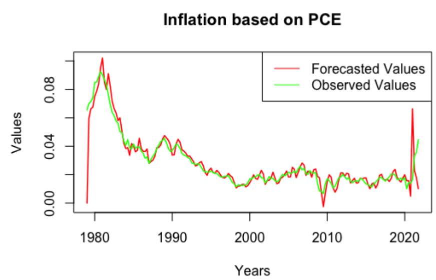
	Inflation (GDP deflator)	**Inflation (GDP deflator)**
	$h = 1$	
RMSE	0.001572535	0.001381618
MAE	0.0002262104	0.0001988875
Hit Ratio	0.9886	0.9886
	$h = 4$	
RMSE	0.007870475	0.007916502
MAE	0.003998073	0.003747319
Hit Ratio	0.5906	0.5439
	$h = 8$	
RMSE	0.01808777	0.01605877
MAE	0.01100277	0.009211446
Hit Ratio	0.4311	0.4371

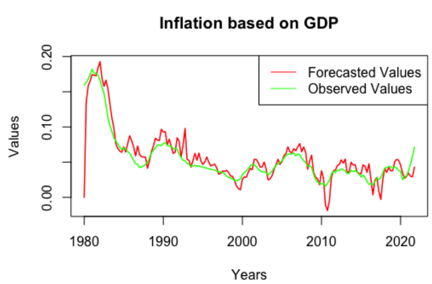
The exercise shows a good overall performance of the model for all forecasts and dependent variables. Focusing on the Hit Ratio, we have rising values as the forecast shrinks, moving from 40% to even 90% of corrected previsions of variable changes for the shorter horizon. Moreover, we present graphics for adherence of the predicted time series of inflation and the actual observed time series, respectively for horizons $h = 1$, $h = 4$, $h = 8$.

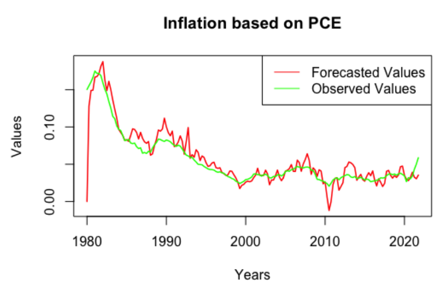












3.3 Expected Number of Predictors

The analysis of the forecast performance consists, furthermore, in the investigation of the average number of regressors in all different models K and of the most influential variables that explain inflation. A measure of how much a variable is important or weights in the forecast exercise is deployed. We evaluate the expected or average number of predictors used in the DMA process at time t as :

$$E(size_t) = \sum_{k=1}^{K} \pi_{t|t-1,k} Size_{k,t} \tag{25}$$

where $Size_{k,t}$ denotes the number of predictors in model k at time t. Here are the graphics for `INFL_GDP` and `INFL_PCE`:

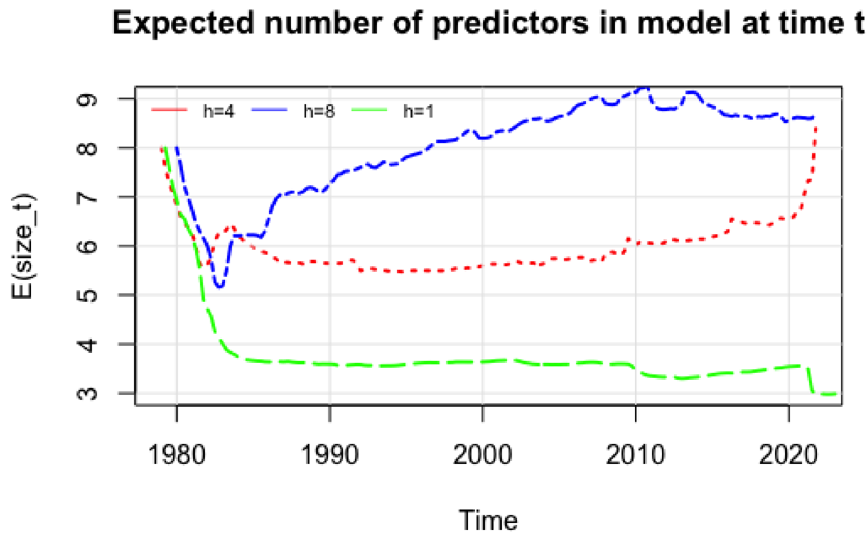


Figure 1: *

Inflation w/ GDP deflator

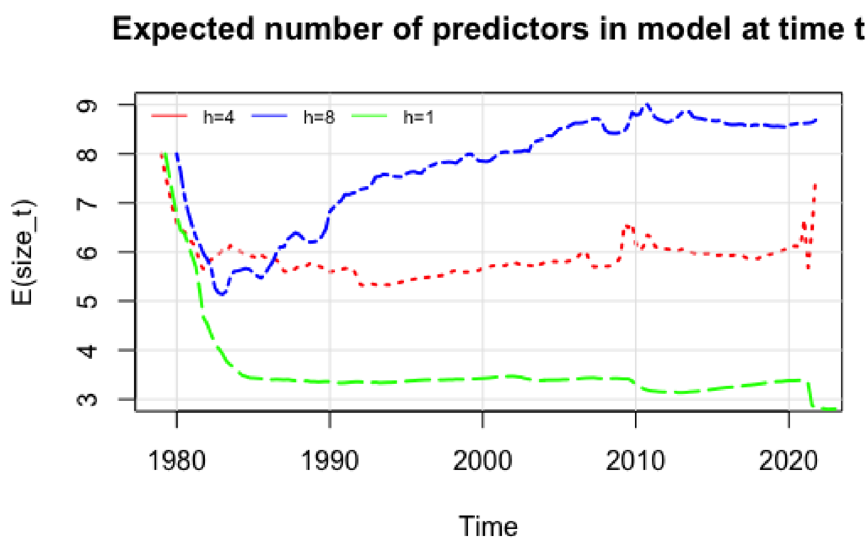


Figure 2: *

Inflation w/ PCE deflator

The first graph refers to the forecast of y_t as the inflation index deflated with the GDP and the second as the inflation index deflated with the PCE. They present a similar pattern: the average number of predictors in the regression increases as the horizon of forecast increases. For $h = 1$ the expected number of regressors is four, for $h = 4$ it is six and for $h = 8$ about eight/nine. This can be interpreted as a logic connection between number of explanatory variables and the width of the forecast horizon: as we try to predict inflation longer in the future, the amount of information needed increases and more variables can enter the model and bring their contribute in the clarification of changes of inflation during times.

3.4 What variables are useful to describe inflation ?

In light of the optimal number of predictors in the model, the next step is to attempt a selection of the most useful variables when forecasting inflation, in each period of time. The first step consists in the analysis of the posterior inclusion probabilities, to see on average which variables were more significant in the prediction model. Table 3 presents the average posterior inclusion probabilities for all variables in the model for each dependent variable and forecast horizon. Figures 3.1-3.6 present a graphical representation of the different influences of these macroeconomic variables on inflation rates. By posterior inclusion probabilities we refer to the probability that a predictor is useful for forecasting at time t.

Table 3: Posterior Inclusion Probabilities

	h = 1		h = 4		h = 8	
Variable	INFL_GDP	INFL_PCE	INFL_GDP	INFL_PCE	INFL_GDP	INFL_PCE
SPREAD	0.1437375	0.116431	0.247086	0.247091	0.291399	0.285674
TBILL	0.174800	0.143832	0.285986	0.311713	0.357960	0.333428
PGDP1* / PCONX1**	0.973124	0.980189	0.925301	0.9041952	0.889225	0.864035
PGDP2* / PCONX2**	0.126030	0.136273	0.647644	0.630985	0.564057	0.532267
UNEMP	0.133604	0.117331	0.269669	0.271860	0.322366	0.314799
PMI	0.136272	0.128359	0.2368015	0.227646	0.249859	0.2495415
HSTARTS	0.077519	0.060499	0.235860	0.260155	0.335984	0.335574
CONS	0.137073	0.116774	0.303123	0.295784	0.248965	0.239359
INV	0.137869	0.129777	0.250364	0.248255	0.273382	0.257662
GDP	0.069455	0.047869	0.263760	0.264948	0.232363	0.229694
EMPLOY	0.170936	0.158481	0.323504	0.319462	0.293594	0.285639
MONEY	0.271388	0.263587	0.392798	0.403366	0.273787	0.272430
INFEXP	0.095402	0.073694	0.253893	0.276556	0.319757	0.327714
CO2EMIS	0.098883	0.076588	0.273182	0.301561	0.362064	0.383697

Table 4: *

*Variables PGDP1 and PGDP2 refer to the lagged value (lag 1 and lag 2) of INFL_GDP: Inflation based on GDP deflator. **Variables PCONX1 and PCONX2 refer to the lagged value (lag 1 and lag 2) of INFL_PCE: Inflation based on PCE deflator.

From this analysis, it results that in general all probabilities increase when the forecast horizon increases. This support our precedent interpretation: in shorter horizons there are less predictors and they are less informative, while for longer horizons, more variables become relevant and their weights increase. The most significant variable in all exercises is the lagged time series of inflation. This supports those theories saying that to predict inflation only its past value is needed. Therefore, as Scacciavillani (1994) and others after, we can conclude that inflation is in fact a process with memory. Moreover, it can be interpreted as a signal of the goodness of the model because it is capable of recognizing and selecting the most determinant variables from the set to influence the movements of the dependent variable. Furthermore, it is notable that for many variables, their relation with inflation rates can change when passing from short to long horizon of forecast, meaning that there are some variable who influence inflation in the short term but not in the long term and viceversa. In this analysis exercise, we found for example that INV (Private Domestic Investments) and GDP are higher for forecast at $h = 4$ than $h = 8$. Also, the addition of CO2EMIS produces some notable results, but details on this topic will be analysed on 3.5. For each horizon, the appropriate average number of variables has been selected to include those with the highest posterior inclusion probability for each exercise. Specifically, we do not include the lagged values of inflation since it is fact-checked their influence. Here we provide a graphic representation.

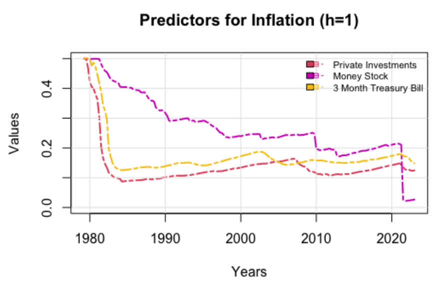


Figure 3: ***Inflation (GDP)***

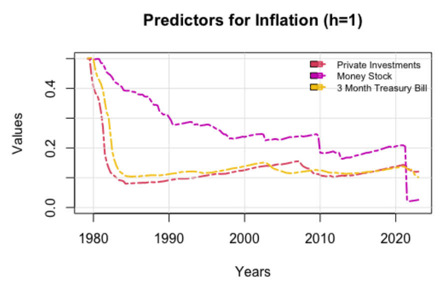


Figure 4: ***Inflation (PCE)***

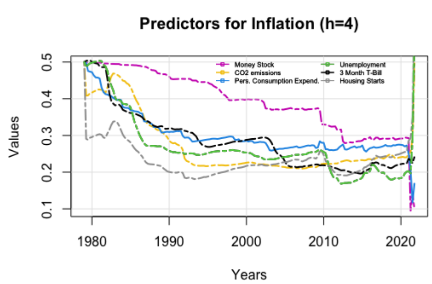


Figure 5: ***Inflation (GDP)***

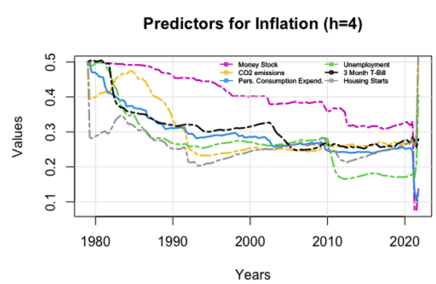


Figure 6: ***Inflation (PCE)***

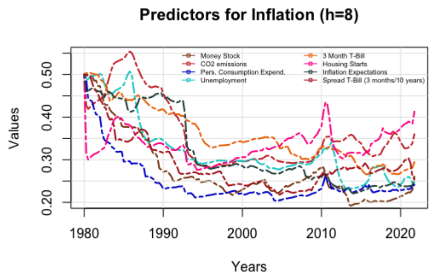


Figure 7: ***Inflation (GDP)***

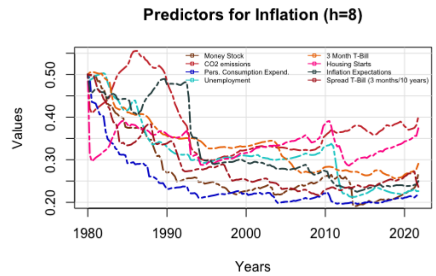


Figure 8: ***Inflation (PCE)***

A few considerations about these results are the following:

- DMA model appears to be able to detect both a slowly but constant influence of a variable and more sharp increases and decreases;
- DMA exhibits that inflation is driven by multiple factors, with some being more influential on short horizons and others acquiring more influence in the long run;
- Inflation, from this analysis, seems to truly follow a time-varying parameters model;
- the final set of explanatory variables reduces of half of potential candidates: Money Stock, CO2 emissions, Inflation Expectations, 3-Month T-Bill, Housing Starts, Personal Consumption Expenditures, Lagged Inflation, Unemployment.

Years ranging from 2020Q1 to 2022Q1 present, in terms of the forecasting exercise, a remarkable challenge. Given the extraordinary nature of the events occurred in these years, the model is still capable to reflect movements of the real word: an increase in the rate of unemployment, a reduction of family expenses given by the reduction of power purchase, an increase of interest rates.

3.5 CO2 Emissions in the model

Here, is discussed the role of the variable "CO2emis" in the model, to see if it has a role or an impact on the movements and the forecasts on inflation rate. From table 2 we can read the posterior inclusion probabilities of this variable in both models and different forecast horizons. Here there is a visual representation of variable "CO2emis" in both evaluations and for each forecasting horizon:

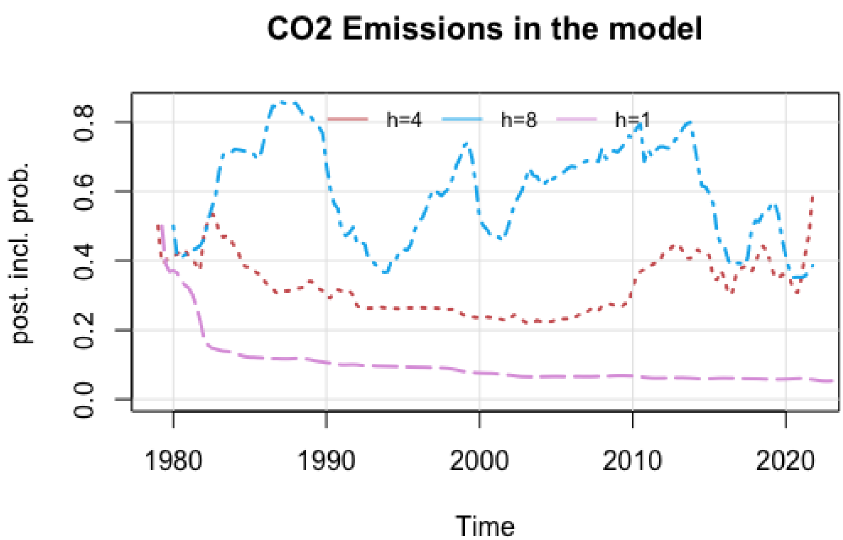


Figure 9: Inflation w/ GDP deflator

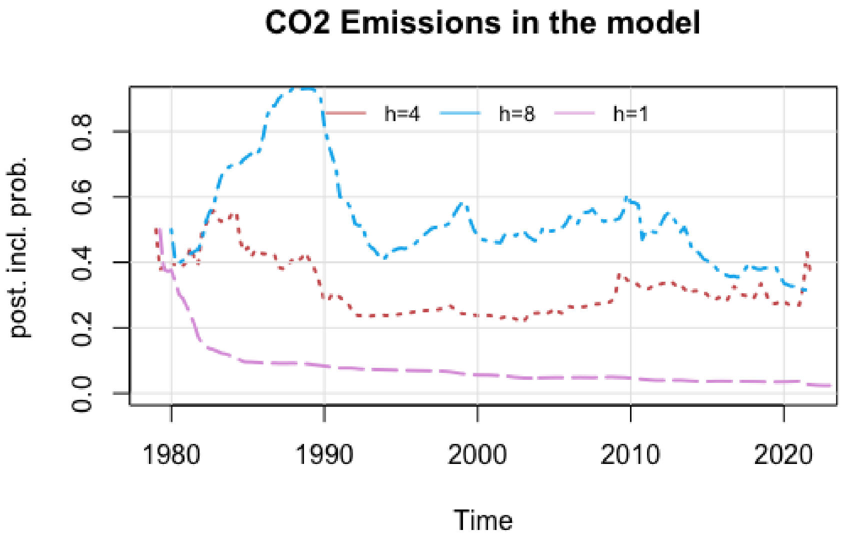


Figure 10: Inflation w/ PCE deflator

The graphs show how the importance of the variable in the model is particularly higher for forecast $h = 8$, reaching various highs in periods 1982-1990; 1996-2000 and 2002-2012. With regard to horizon $h = 4$ and $h = 1$ its impact on the short run is downsized, the only increase detected is for one of the same periods yet cited (2010-2020). There can be many different interpretations for these movements. In the 1950's the issues regarding climate changing started to arise from several scientists and scientific studies. But it wasn't until the 1980's that all scientific community begin to unite and warn countries' leader and international organizations to the dangers of human's print on earth and demand for actions to prevent further damages to out planet. From there on, population is acquiring knowledge of the problem and demanding for action. Entering the new millennium, also central banks and international institutions begin to include climate-related data in their monetary policy work formation. From figures 9 and 10, it seems that its impact is stronger in the long run. Long run, in this case, expresses the future, one for whom we would all be hopeful for. Thus, it is essential to consider the footprint of a country in the environment to properly consider the augmented costs in terms of prices, fueling demand and supply chains.

4 Conclusions

This work has expanded the work of Koop et al. (2012), investigating two main queries. First, the model has been tested with a new and more recent data-set. The evidence continues to support the time-varying nature of the variable and the adherence of the model to the study. Second, we have included CO2 emissions in the set of variables used to forecast the Phillips curve. We found a significant relation between inflation and climate changes meaning that a measure of pollution (more generally any climate risk indicator) should be always taken into consideration to forecast the inflation. The linkage found between macroeconomics and CO2 emissions may help policy makers to restore financial stability while guaranteeing future generations with the planet Earth that preserves the same possibilities that were given to the older generations.

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