Dynamic Time Warping Clustering for diversified portfolios: an empirical analysis

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Abstract

The Sharpe ratio is used to measure risk-adjusted returns for traded securities and it is quite useful to analyze long-term investments. It is one of the measures used by fund managers in the assessment of investment performance. In this guise, we assume it could be a key indicator to group securities that show similar dynamics of the Sharpe ratio over time. We use a hierarchical clustering analysis to identify groups of securities from the S&P500, which show similar dynamics of the Sharpe ratio during the last decade. Using a Dynamic Time Warping method, we identify two different clusters. To investigate how different Sharpe ratio dynamics may impact on portfolios' diversification, we use the Most Diversified Portfolio approach (Choueifaty and Coignard, 2008) to evaluate the gain in terms of diversification and provide some insights.

Keywords: Clustering; Portfolio selection; Diversification.

1 Introduction

Security selection and asset allocation have been a challenge for financial scholars and practitioners.

In 1952, in his "Portfolio Selection" a young Harry Markowitz pioneered his theory, for which he was later awarded a Nobel Prize, and that is still today like a lighthouse in the search for the best asset allocation. Since then, data mining and machine learning have significantly transformed the finance industry with algorithmic trading, novel methodologies for credit scoring, and new ways for portfolio selection and optimization. In this last case, a popular "feature" to make clusters is represented by correlation (or, however, correlation-based metrics) (e.g. Sass and Thös, 2021), or an alternative solution is simply using the returns (e.g. Gubu et al., 2019). Using clustering techniques to build groups of stocks which show similar features with the aim to provide a tool for portfolio selection is becoming quite common among asset managers. When it comes to portfolio's weights it is well known that the market capitalization–weighted indices are not efficient. Various alternative solutions can be found, such as fundamental indexation and equally weighted portfolios; we want to investigate how clusters of stocks which have similar dynamics of the excess return- risk tradeoff may provide an alternative way of portfolio selection aiming to get a better performance. In this work we choose to measure each security's excess return using the Sharpe Ratio and assume that its dynamics over time may represent a valid criterion to identify similar volatility structures.

To the best of our knowledge, the analysis of Sharpe Ratios' time series for clustering purposes represents a novelty in literature and industry. The Sharpe Ratio Index was developed by William F. Sharpe (Sharpe, W. F., 1966), who referred to his measure as "reward to variability", recalling the embedded elements that compose it, and that allows capturing both returns and risk as a whole. We applied a hierarchical clustering analysis using securities constituents the S&P500 to identify clusters with similar dynamics of the Sharpe Ratio. Using a Dynamic Time Warping (Sakoe and Chiba, 1978) method, and four Cluster Validation Indices we identify two clusters one containing 215 securities and the other 251. We estimate the average weekly Sharpe Ratios in each cluster, which result non stationary (each series is $I(1)$), to compare their dynamics we investigate the existence of cointegration between the two series using the Engle & Granger cointegration test. We find that the two variables are not cointegrated so the two series are not affected by a common driver. We also analyze the main features of the securities belonging to each cluster, and to investigate possible implications on correlations among assets' returns, we use the Maximum Diversified Portfolio (MDP) approach by Choueifaty and Coignard (2008), on equally-sized samples selected from both clusters and from each cluster.

The remainder of this paper is organized as follows: Section 2 reports the state of the art. Section 3 outlines the methodology used. Section 4 describes the data and shows the empirical results. Section 5 summarizes the main results and provides some concluding remarks.

2 Literature review

In recent years, using data mining techniques, such as clustering, has become quite common for portfolio management.

There are no real standards, especially since clustering is often generally used as an exploratory analysis, preceding the real one, while in this kind of implementation, this data technique has a key role in decision-making for portfolio selection. Nanda et al. (2010) use K-means algorithms to build Markowitz efficient portfolios and to show the power of diversification derived from clustering. Lu et al. (2018) build a portfolio strategy using modularity, and find that their strategy outperforms the Markowitz portfolio.

Time series clustering is gaining importance in finance, and we can identify three categories of time series clustering: observation-based clustering, model-based clustering, and feature-based clustering.The observation-based clustering relies on raw data, as in Arévalo et al. (2019), in which the time series to identify clusters are represented by the returns. The model-based clustering exploits models fitted on the series, like ARIMA, or GARCH, as in D'Urso et al. (2013) and D'Urso et al. (2016). The feature-based clustering employs features extracted from the observed time series, as cross-correlation like in Alonso et al. (2019), or a more sophisticated time series clustering based on the estimated cepstrum, which has been proven to be better with respect to other clustering models (D'Urso et al., 2020).

Another class of "similarity metrics" is used in Puerto et al. (2020), who in their one-stage framework (i.e. a programming issue to make clustering and portfolio selection as a whole), propose a criterion based on correlation coefficients between assets' returns. Also Leòn et al. (2017) apply a set of clustering algorithms to demonstrate that portfolio volatilities generated through clustering are smaller when compared to the portfolio obtained using classical Mean-Variance Optimization (MVO) overall in the dataset. Furthermore, there are even contaminations from other disciplines, for example, Cheong et al. (2017) proposed a multistage portfolio optimization for active portfolio management based on clustering, in which the used algorithm was borrowed from genetics to express investor information.

Our time-series can be classified in the feature-based clustering techniques, as we use the Sharpe Ratio variations as a "synthetic" indicator of the embedded features in stocks, and as measure of similarity/dissimilarity among elements when clustering. The focus of our analysis is to investigate wheteher different Sharpe Ratio dynamics could have an effect on different levels of portfolio diversification.

In our analysis, diversification has a key role, being the purpose of clustering analysis. There is not a unique definition in literature for diversification; indeed, if from a qualitative point of view it deals with the opportunity to mitigate the effect of specific risk *not having all the eggs in one basket*, when we want to assume a quantitative point of view, it can be expressed by a variety of portfolio strategies (Fragkiskos A., 2014), acting on number of assets (Sass and Thös, 2021), weights (Bouchad et al., 1997), return (Chambers and Zdanowicz, 2014), risk ratio (Tasche, 2006), among others.

To quantify diversification, we recall a measure based on Tasche's formalization (Tasche, 2006), and we use the Most Diversified Portfolio approach introduced by Choueifaty and Coignard (2008), which optimizes portfolio Diversification Ratio (Choueifaty, 2006) to build portfolios with a better risk-return trade-off (Choueifaty et al., 2011).

3 Methodology

In this work, we used hierarchical clustering techniques to analyze the time series of the Sharpe Ratios for stocks constituent the S&P500 and identify groups of securities with similar dynamics.

3.1 Clustering Techniques

Clustering is a data analysis technique used to partition a dataset into groups, clusters indeed, of similar elements based on certain features. The main goal of clustering is to identify natural groupings in the data such that elements within the same cluster are more similar to each other than they are to elements of different clusters. Clustering algorithms aim to maximize the homogeneity within clusters while maximizing the heterogeneity between clusters. This objective ensures that each cluster represents a distinct subset of the elements with common characteristics, allowing the identification of patterns, trends, and relationships within homogeneous subsets of data. Simultaneously, by maximizing heterogeneity between clusters, clustering ensures that the identified groups are distinct from each other, capturing the diversity and variability present in the dataset. Therefore, clustering is a powerful tool for segmentation and detecting hidden structures within datasets. Clustering techniques can be categorized into hierarchical and partitional ones; the main difference is that the former builds "nested" clusters, while the latter allowes to obtain not overlapping groups of elements. The choice between them strictly depends on the dataset and, above all, the meaning attributed to clusters. From an operational point of view, partitional clustering (e.g., algorithms like K-means) requires the analyst to specify the number of clusters a priori, so it could be unsuitable when the number of clusters is unknown. Furthermore, you might wish to fully leverage the capability of clustering in searching for not trivial patterns in your data, avoiding misleading assumptions with the explicability of the clusters' number. We use hierarchical clustering and set the optimal number of clusters thanks to the four most common Clustering Validity Indexes (CVIs): Silhouette (Rousseeuw, 1987), which measures how similar an object is to its own cluster, Dunn (Bezdek & Pal, 1988) which evaluates cluster compactness and separation of clusters, Calinski–Harabasz (Calinski–Harabasz, 1974), which measures the ratio of between-to-within cluster dispersion, and COP (Gurrutxaga et al., 2010), which evaluates the level of order preservation in clustering. The optimal number of clusters is obtained maximizing or minimizing the CVIs, precisely maximizing the first three and minimizing the last one.

Using hierarchical techniques is possible to obtain a structure of nested clusters with different levels of granularity instead of a simply flat partition of elements. In this work, we used hierarchical clustering for time series data, which involves the unsupervised process of grouping similar temporal sequences based on their patterns and trends over time. In the case of our analysis we aim to build clusters of stocks which show a similar dynamics of the risk-return tradeoff, precisely we measure it using the Sharpe Ratio of each stock, as defined by Sharpe W.F. (1966):

$$SR_i = \frac{E(R_i) - R_f}{\sigma_i} \tag{1}$$

where $E(R_i)$ is the Expected Return of security, i, R_f is the risk-free rate of return, and σ_i is the standard deviation of the security's return.

3.2 Estimation of distance matrix and clustering algorithms

A key point in clustering techniques is the estimation of similarity among elements using a similarity/dissimilarity measure to evaluate the closeness between each pair of elements. The similarity measures can be categorized into lock-step, elastic, and geometrical. In the first group, the comparison is based on the temporal index (one-to-one comparison between the i-th values of the series). These measures include the Euclidean distance and the correlation-based (cross-correlation) distance. The Euclidean Distance is the result of the sum of the point-to-point distances and, like the other lock-step measures, cannot be used in the case of a time series with different lengths by definition. To overcome this problem, elastic measures are applied to compare misaligned time series. These methods are based on the edit distance, which is the minimum number of insertions, deletions, and substitutions needed to change one string to another and, for that, able to capture temporal distortions. One example of these measures is Dynamic Time Warping (Sakoe and Chiba, 1978), which is also used in this work. Finally, geometry-based Measures use the shape as a geometric feature of the multivariate time series.

3.3 Dynamic Time Warping

The Dynamic Time Warping (DTW) technique was initially developed for speech recognition (Sakoe and Chiba, 1978), but it is now used in a variety of fields. It uses a dynamic programming approach to align the time series finding the optimal warping path under constraints, like monotonicity, continuity, warping window, slope constraint and boundary conditions (Berndt and Clifford, 1994)

Given two series $A = \{a_1, a_2, \ldots, a_{i,\ \ldots}\ , a_T\}$ and $B = \{b_1, b_2, \ldots, b_{j,\ \ldots,}\ b_T\}$, for each point (i, j), DTW recursively minimizes :

$$D(A, B) = d(a_i, b_j) + min \begin{cases} d(a_{i-1}, b_{j-1}) \\ d(a_i, b_{j-1}) \\ d(a_{i-1}, b_j) \end{cases}$$

In this work, our dataset is made up of time series of the same length and speed, so the temptation to rely on a lock-step measure could be strong, even due to the higher computational cost of calculating the distance matrix using the DTW method. However, DTW was revealed to be more sophisticated, taking into account the time-varying features of the series, enabling us to make conclusions not only on how "far" time series are than each other but also to compare them for their variations. Once the distance matrix is obtained, clustering algorithms can be applied.

3.4 Linkage criteria

Four linkage criteria—Complete, Average, Ward.D, and Ward.D2—have been tested to compare hierarchical metrics.In hierarchical clustering, an element is placed in a subset or another by considering the similarity/distance between single elements and generalizing it to the distance between clusters of elements (intergroups dissimilarity). Thus, the similarity matrix considers similarities (distances) between couples of elements, while the linkage metric is built considering distances through clusters. The average linkage, also known as minimum variance method, the distance between two clusters is determined by the average distance from any member of one cluster to any member of the other cluster (Saxena et al., 2017); it is defined as: $\frac{1}{|A||B|}\sum_{a\in A}\sum_{b\in B} d(a,b)$. The complete linkage, also called the diameter, the maximum method or the furthest neighbor method, the distance between two clusters is determined by longest distance from any member of one cluster to any member of the other cluster (Saxena et al., 2017); it is defined as: $max\,\{d(a,b) : a \in A, b \in B\}$ In Ward's linkage (Ward, 1963) the partition is obtained by finding the optimal value of an objective function, that can be 'any function that reflects the investigator's purpose', and in the example provided by the author, the objective function is represented by the error sum of squares, that is why this method is also know as'Ward's minimum variance method'; it has been defined as: $2 * \frac{|A||B|}{|A|+|B|} * d\left(\frac{\sum_{a\epsilon A} a}{|A|}, \frac{\sum_{b\epsilon B} b}{|B|}\right)^2$ or in its variant WardD2: $\sqrt{2 * \frac{|A||B|}{|A|+|B|}} * d\left(\frac{\sum_{a\epsilon A} a}{|A|}, \frac{\sum_{b\epsilon B} b}{|B|}\right)^2$.

3.5 The Diversification Ratio

In the portfolio selection process a key element is represented by the diversification effect which may generate better portfolio performance in terms of risk-return trade-off. To quantify diversification Tasche (2006) proposes the following measure:

$$DF_{\varrho}(X_i|Y) = \frac{\varrho(X_i|Y)}{\varrho(X_i)}$$

whereX_1,..., X_n are real-valued random variables and $Y = \sum_{i=1}^{n} X_i$, and ϱ is a risk measure such that $\varrho(Y), \varrho(X_1)$,..., $\varrho(X_n)$ are defined. $DF(\bullet)$ denotes the marginal diversification factor of subportfolio X_i with respect to the risk measure ϱ. We then use the Most Diversified Portfolio approach introduced by Choueifaty and Coignard (2008), which optimizes portfolio the Diversification Ratio (Choueifaty, 2006) allowing to build portfolios with a better risk-return trade-off (Choueifaty et al., 2011).

$$DR(w) = \frac{\sum_i (w_i \sigma_i)}{\sqrt{w' \Sigma w}} \tag{2}$$

where X_1,..., X_n are stocks with Expected returns $\mu = (\mu_i)$, volatility $\sigma = (\sigma_i)$, correlation matrix $C = (\rho_{i,j})$, and covariance matrix $\Sigma = (\rho_{i,j}\sigma_i\sigma_j)$; $w = (w_i)$ are the weights of a long-only portfolio,

$\sigma(w) = w'\Sigma w$ its volatility, and $\langle w|\sigma\rangle = \sum_i w_i\sigma_i$ its average volatility: $DR(w)$ is defined as the ratio of the portfolio's weighted average of volatility to its overall volatility . Choueifaty and Coignard (2008) proposed a Diversification Ratio decomposition:

$$DR(w) = [\rho(w)(1 - CR(w)) + (CR(w)]^{-1/2} \tag{3}$$

in which $\rho(w)$ is the volatility-weighted average correlation of the assets in portfolio,

$$\rho(w) = \frac{\sum_{i\neq j}(w_i\sigma_i w_j\sigma_j)\rho_{i,j}}{\sum_{i\neq j}(w_i\sigma_i w_j\sigma_j)} \tag{4}$$

and $CR(w)$ is the volatility-weighted Concentration Ratio (CR) of the portfolio:

$$CR(w) = \frac{\sum_i(w_i\sigma_i)^2}{(\sum_i w_i\sigma_i)^2} \tag{5}$$

The Most Diversified Portfolio (Choueifaty and Coignard, 2008) can be obtained maximizing the *DR(w)*:

$$w^{MDP} = argMaxDR(w)_{w\in\Pi} \tag{6}$$

where $\Pi^+$is the set of long-only portfolios with $w * I = 1$.

4 Data and results

We use daily stock prices (Yahoo Finance), for the 466 stocks constituent the Standard & Poor 500 Index. We estimate weekly returns to reduce the noise of returns' dynamics (from Wednesday to Wednesday[1]) . The stocks selected were the only ones which have been listed in the index for the 10-year period (2 January, 2014 - 27 December, 2023) which counts 522 weekly stock returns: $R_t = ln\big[\frac{P_{t+7}}{P_t}\big]$. We estimate yearly returns and volatilities using historical rolling window of 52 weeks:

$$E(R_t) = \frac{\sum\limits_{t=1}^{t+51} R_t}{52} * 52 \tag{7}$$

$$\sigma_t = \sqrt{\frac{\sum_{t=1}^{t+51} R_t^2}{52}} * \sqrt{52} \tag{8}$$

and the yearly Sharpe Ratios for each stock:

$$SR_t = \frac{E(R_t) - R_f}{\sigma_t} \tag{9}$$

We use the DTW clustering method to identify common dynamics of the Sharpe Ratio (SR_t) to detect groups of securities which show similar dynamics of the risk-return trade-off. We implement the cluster analysis using the *TSclust* R package (Montero and Vilar, 2014) and the DTW R package developed by Giorgino (2009).

To find the optimal numbers of clusters, we compare the four CVIs according to different linkage criteria varying between 2 and 5, we report the four CVIs in Figure 1.

[1]It was a way not to have the "weekend-effect"; only in three cases, Wednesdays data were not available, e.g. for National Festivity, and were replaced by the Thursday data, not to have a two-weeks gap.

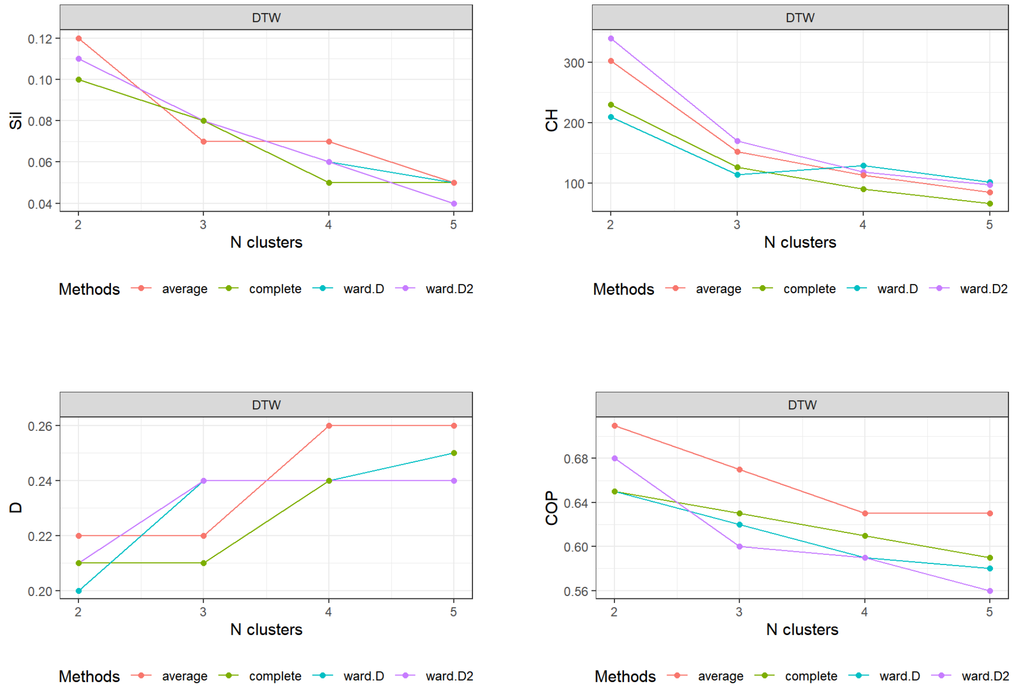

Figure 1: CVIs for 2≤k ≤ 5.

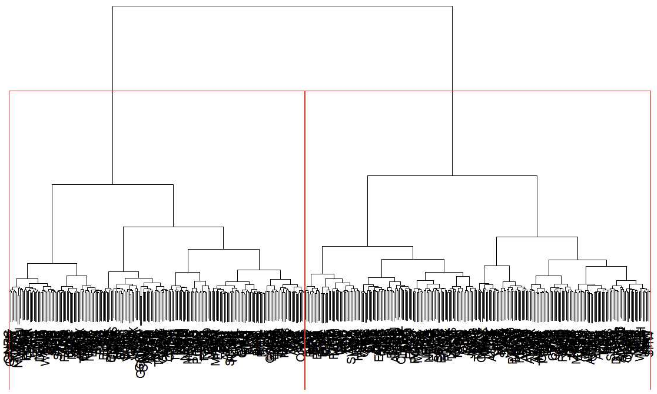

Figure 2: Cluster Dendrogram for k=2 and Ward.D Linkage criterion.

The Silhouette and CH Index point k=2 as the optimal number of clusters based on all of the linkage criteria, the other CVIs provide a number of clusters different according to each linkage criterion. We then build the dendrogram (Fig. 2) for each linkage criterion and we find that the optimal number of clusters is 2 on the basis of the Ward.D linkage criterion.

The two clusters C_1 and C_2 contain respectively 215 and 251 securities. In Figures 3 and 4 we report the average weekly returns and weekly volatilities estimated over the securities constituent each cluster.

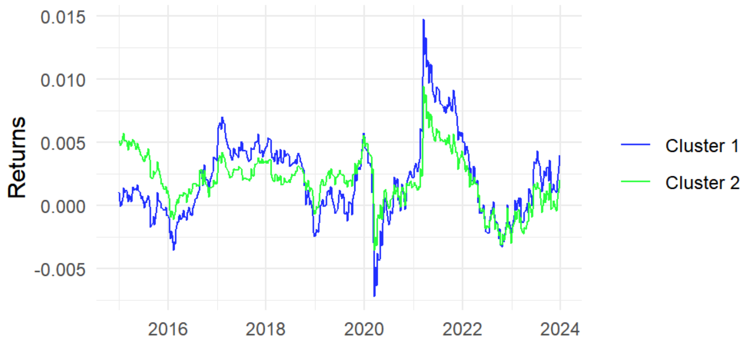

Figure 3: Average weekly returns' dynamics in the two clusters.

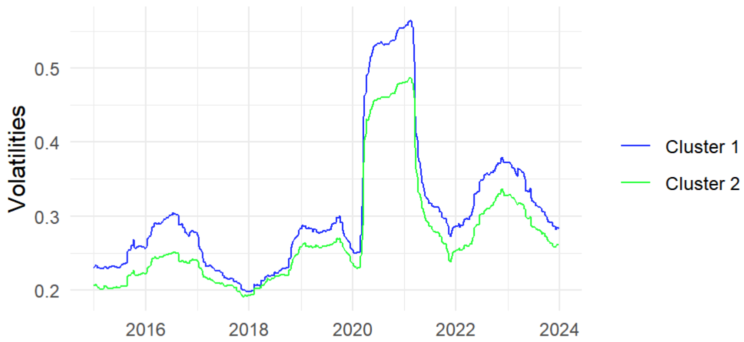

Figure 4: Average weekly volatilities' dynamics in the two clusters.

Metric	Cluster	N	min	median	mean	max	SD
Returns	1	215	-0.0072	0.0015	0.0021	0.0148	0.0032
	2	251	-0.0036	0.0020	0.0020	0.0095	0.0021
Volatility	1	215	0.1973	0.2851	0.3069	0.5648	0.0923
	2	251	0.1904	0.2541	0.2731	0.4869	0.0763

Table 1: Returns and volatilities summary statistics in the two clusters.

In Table 1 we report the descriptive statistics of the two clusters in terms of weekly returns and weekly volatilities. It is worth to notice that average returns are quite volatile: at the beginning of 2015 Cluster 2 shows higher average returns than Cluster 1, they revert after 2020; average volatilities show a clear pattern with Cluster 2 showing always lower volatility.

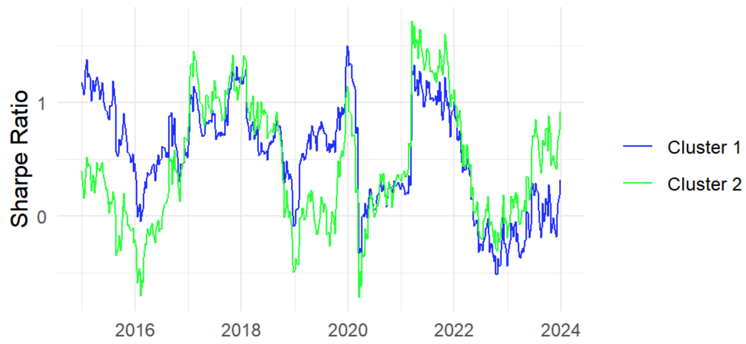

Figure 5: Expected Sharpe Ratio dynamics by cluster.

In Figure 5 we report the dynamics of the average Sharpe Ratios ($E(SR_t)$) by cluster, we observe a non-stationary dynamics of the two series (we tested for Unit Root and both series result $I(1)$). We aim to analyze the difference between the two series using statistical methodologies; in case of non-stationary time series, the simplest approach is to test for cointegration. If two time series, each $I(1)$, result cointegrated it means that they share a common source of randomness. We use the Engle-Granger two-way test (Engle and Granger, 1991) to compare the two Sharpe Ratios.We test for stationarity of $z_{t1}$and z_{t2} in the following relationships:

$$E(SR_{t,C_1}) = \alpha + \beta * E(SR_{t,C_2}) + z_1 \tag{10}$$

and

$$E(SR_{t,C_2}) = \alpha + \beta * E(SR_{t,C_1}) + z_2 \tag{11}$$

Assuming $z_1$and z_2 are $I(1)(\mathrm{H}_0)$, we can not reject the null hypothesis ($I(1)$), concluding that the two series are not cointegrated.

	Statistics	p-value
z_1	-2.2476	0.4733
z_2	-3.4717	0.0451

Table 2: Results of ADF Test on residuals of linear regression models ($\alpha = 0.05$).

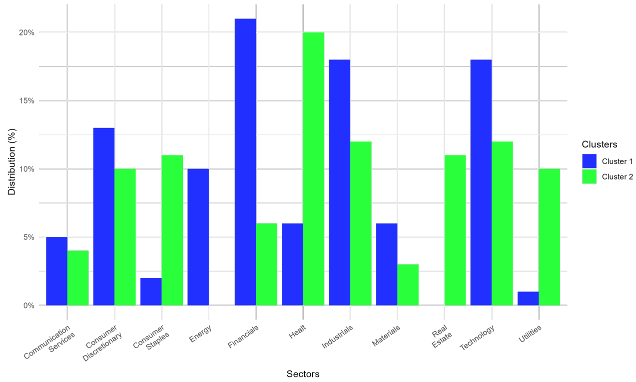

Figure 6: Bar plot of sectors by cluster.

It is interesting to notice the distribution of securities among different sectors in each cluster (Fig. 6); Cluster 1 indeed, presents a concentration of companies mostly belonging to Financials, Consumer Discretionary, Industrial and Technology (that together cover more than the 70%), while in Cluster 2 sectors like Health (20%), Real Estate (11%) and Utilities (10%) together cover more than the 40%. There are no Real Estate companies in Cluster 1, while in Cluster 2 there are no companies belonging to the Energy sector. This shows that we were able to capture specific dynamics of the excess return/volatility trade-off existing in companies belonging to different economic sectors.

Given that the two clusters show structural differences in the Sharpe Ratios, we expect to have quite different behaviour of volatilities and expected returns. Following standard portfolio literature (Markowitz), diversification benefits should be obtained building a portfolio containing securities chosen from the two clusters. To prove this we choose to compare the performance of portfolios selected from the two clusters and from each cluster.

We build portfolios composed by all equal number of securities from each cluster; assuming to select as size of portfolio the 20% of the securities composing each cluster, we have to select 40 securities from each cluster and build a portfolio with 80 securities; then we compare this portfolio with those built with 80 securities from each cluster.

We draw 1000 samples of N=80 securities for each of the portfolios to compare, and we build the portfolios using the Most Diversified Portfolios approach (Choueifaty and Coignard, 2008). We report the results of only one of the samples, by way of illustration.

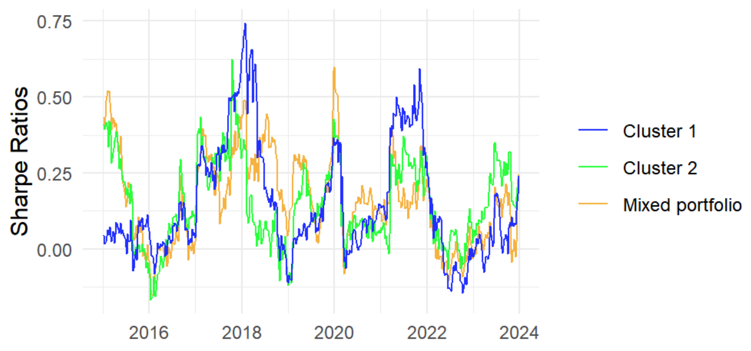

Figure 7: Example of portfolios' Sharpe Ratios with N=80 securities for each cluster and for the mixed portfolio.

In Figure 7, we see that the portfolio built with securities from both clusters does not provide a better portfolio's performance, compared to portfolios built on each cluster.

By observing the different distribution of securities among different sectors in each cluster (Fig. 6), we may hypothesize that the performance of each security could be affected by the sector to which it belongs. In order to evaluate if the distribution of securities among sectors may have an impact, we build mixed portfolios with securities drawn from both clusters, considering an equal distribution of the securities among sectors in each drawn sample, and then we compare it with the portfolios from each cluster.

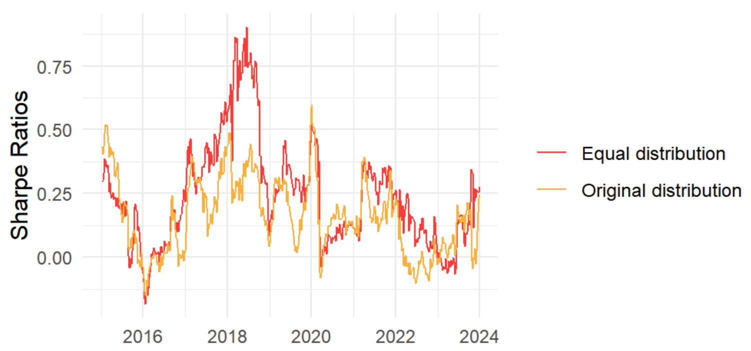

Figure 8: Example of portfolios' Sharpe Ratios with N=80 securities in mixed portfolios according to the original distribution and the equal distribution.

As we see in Figure 8, the Sharpe Ratio in mixed portfolios built according to the equal distribution of securities among sectors seems to be higher than the Sharpe Ratio obtained for the portfolios with the original distribution. To evaluate if the mixed portfolios, when taking into account the distribution of securities among sectors, is preferable, we now compare the mixed portfolios according to both the original and the equal distribution among sectors in clusters, and the portfolios on each cluster.

Figures 9a-b and 10a-b show the results.

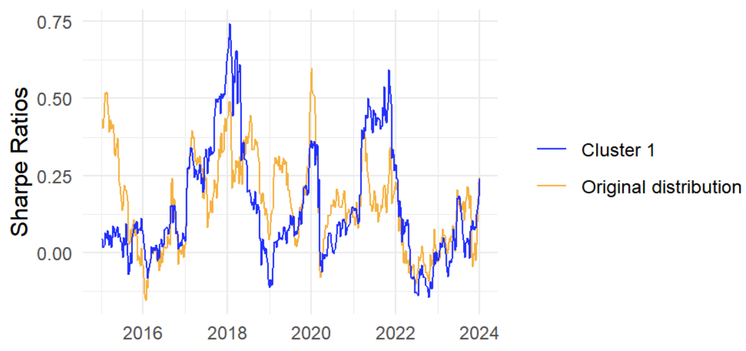

Figure 9a: Example of portfolios' Sharpe Ratios with N=80 securities in mixed portfolio according to the original distribution vs. the portfolio built from Cluster 1.

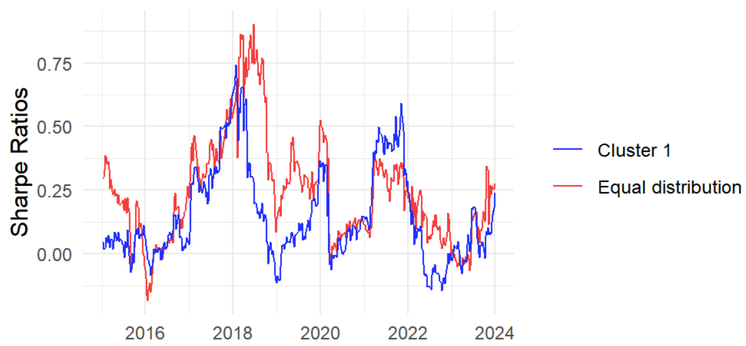

Figure 9b: Example of portfolios' Sharpe Ratios with N=80 securities in mixed portfolio according to the equal distribution vs. the portfolio built from Cluster 1.

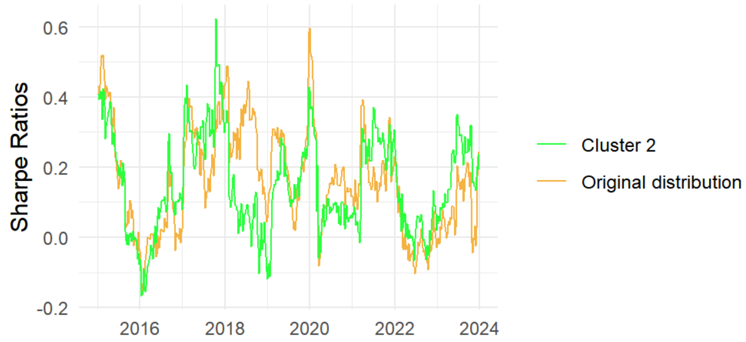

Figure 10a: Example of portfolios' Sharpe Ratios with N=80 securities in mixed portfolio according to the original distribution vs. the portfolio built from Cluster 2.

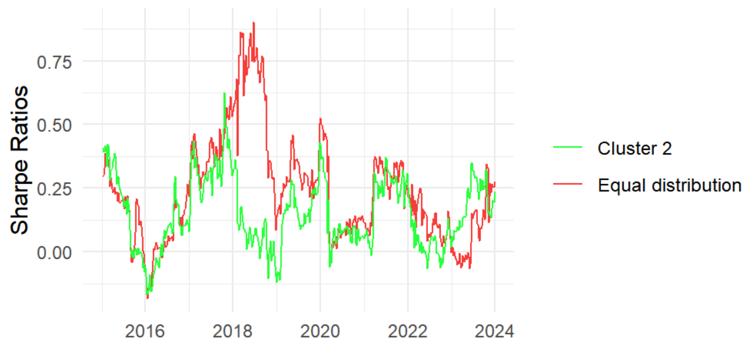

Figure 10b: Example of portfolios' Sharpe Ratios with N=80 securities in mixed portfolio according to the equal distribution vs. the portfolio built from Cluster 2.

In Figures 9a-b and 10a-b we see that the mixed portfolio seems to have better performances than both Cluster 1 and Cluster 2, when we take into account the distribution of securities among sectors in clusters, showing that this one have a high impact on clusters.

5 Conclusions

Due to the usefulness of the Sharpe Ratio in evaluating long-term investments, we assume to use it as an indicator of"similarity" between securities. Using hierarchical clustering analysis and Dynamic

Time Warping methodology, we have a partition of the stocks constituent the S&P500 into two groups, showing distinct Sharpe Ratio dynamics (Fig. 5). We build the Most Diversified Portfolios (Choueifaty and Coignard, 2008) for N=80 stocks randomly selected from both clusters and from each cluster. The aim of the work was to investigate whether a Sharpe Ratio dynamics'-based clustering results effective in maximizing portfolio performances.

We find that the clustering analysis splits the securities' universe in two different groups, where Cluster 2 seems to be less volatile than Cluster 1 (Fig. 4) and we also find that the two clusters show a different distribution of securities among sectors (Fig. 6). We find that this one has an impact on clusters, as we obtain a different result in the mixed portfolios from both clusters, when it has been taken into account or not (Fig. 8), showing that when it has been considered, the mixed portfolio results better than the ones drawn from each cluster (Fig. 9a-b and 10a-b).

This work opens the way for future research, as a lot of aspects have to be deepen, in various directions, for example alternatives clustering techniques could be used, and also an approach like in Li et al. (2021), using DTW graph-based clustering could be tested, to make a comparison on the effects.

As we find that the two clusters dynamics are not cointegrated, a further research will be a Principal Component Analysis with the aim to analyze the different risk drivers that could explain the different volatility in the two clusters (Fig. 4).

Then, we chose an optimal number of clusters equal to two, according to the dendrogram (Fig. 2) and to two out of four the CVIs considered (Fig. 1). We aim to repeat the analysis, following the other two CVIs pointing an optimal number of clusters equal to five, and make a comparison of the results.

Finally, in this work the Sharpe Ratio has a key role, as we use it with a double aim: *ex-ante*, as a tool for the clustering analysis, in order to evaluate if the Sharpe Ratio variations could be assumed to identify structural feature of securities, and then, *ex-post*, as a portfolio's performance indicator, as usual. A forward step could be represented by the use of the Information Ratio rather than the Sharpe Ratio, as the former, taking into account a benchmark index, could detect the structural dynamics adjusted for the dominant features.

6 References

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