Measuring financial sustainability and social adequacy of the Italian NDC pension system under the COVID-19 pandemic

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Abstract

According to the International Monetary Fund [9], the COVID-19 pandemic is currently affecting both financial sustainability and the social adequacy of public pension systems. In this paper, we measure the effects of the pandemic on the Italian public pension system by modeling the evolution of some key variables, such as unemployment rate, wage growth rate, inflation rate, and mortality rates, which are involved in the evaluation of the pension system from both the side of contributions and benefits amount, considering or not the shock due to the pandemic. Our analysis shows that the Italian system seems to be resilient in the long run to financial stress, however, showing a critical evolution of social adequacy.

keywords: Notional Defined Contribution pension systems, COVID-19, Social adequacy, Financial sustainability.

1 Introduction

On March 11, 2020, the coronavirus disease (COVID-19) outbreak has been declared a pandemic by the World Health Organization (WHO [19]). Most of the countries worldwide have introduced restrictive measures to contrast the spread of the virus. If on one hand, the containment measures imposed by governments have partially limited the increase in infections and saved many lives, on the other, they have seriously damaged the economic activities. The closure of many production activities caused significant losses for the majority of the economic sectors. Many people have been directly affected by significant economic losses, suffering wages cut due to the temporary closure of the work activities. While the decrease in sales and the stop of production implied revenue losses for many companies.

The International Monetary Fund [9] argues that the COVID-19 pandemic is impacting both the financial sustainability and the social adequacy of public pension systems. The restrictions on economic activity associated with the pandemic are affecting the labor market by reducing employment and stagnating or deteriorating real wages, thus probably lowering contributions paid by both employees and employers. Specifically, employers could be unable to honor their contractual obligations, then becoming delinquent in their contributions and reducing the pension system revenue. The other source of contribution to the pension system are the investment gains, also decreasing due to the financial crisis following the ongoing COVID-19 pandemic. In funded pension schemes, lower returns reduce their asset values, then exposing the pension system to a fiscal risk. These developments, which make uncertain the state's budget, could influence the capacity of governments to maintain the solvency of the public pension system under adverse conditions.

The aim of our paper is analyzing the effects of COVID-19 pandemic on the financial sustainability and adequacy of the pension benefits to retirees of Notional Defined Contribution (NDC) pension systems. The NDC pension system is founded on a pay-as-you-go (PAYG) mechanism, where total contributions are used to pay the pensions of current pensioners.While the pension formula follows a defined contribution scheme with the accumulation of contributions during the working age and until the retirement age in a individual account. At retirement, the accumulated contributions plus the earnings from a virtual rate of return usually tied to economy is converted into a whole life annuity (or pension amount). There is no asset set aside. The attribute notional is due to the PAYG funding mechanism involving the institution of virtual or notional accounts which are the basis for the benefits calculation. The key variables of such a system are the contribution rate which is fixed, the virtual rate of return or notional rate, the retirement age, the benefits indexation and the conversion factor which transforms the notional account into the pension benefit. Examples of NDC introduction are Sweden (1994), Italy (1995), Latvia (1996) and Poland (1999) (Chłoń-Domińczak et al. [3]).

The main appealing features of NDCs are the individual actuarial fairness as contributions are directly linked to pension benefits and the long-term financial sustainability favored by a defined contribution nature (Palmer [14]). In theory, these systems are in equilibrium under constant demographical and economic conditions (steady state), however longevity improvements, aging populations and fertility decreases, as well as worse economic and labor conditions, compromise the financial sustainability and the guarantee of adequate (in terms of living standard) pensions to retirees. This latter condition is defined as social adequacy.

Aiming at addressing the effects of COVID-19 on the financial sustainability and social adequacy of a NDC pension system, we model the impact of the pandemic as a deterministic shock affecting the unemployment rate, wage growth rate, inflation rate and survivor rates, which are involved in the calculation of the pension system from both the side of contributions and benefits amount. All these variables are modeled as stochastic.

The paper is organized as follows. Section 2 introduces the fundamentals of the NDC system. Section 3 describes the models for the economic variables involved in the COVID-19 pandemic crisis. Section 4 concerns the measurement of social adequacy and financial sustainability of the system. Section 5 illustrates the numerical application considering the Italian NDC system. Finally, Section 6 provides the conclusions.

2 The fundamentals of the NDC system

This section provides the fundamentals of the NDC system. Particular attention is devoted to the description of how populations (active and beneficiary), contribution and benefits evolve over time. Finally, the main characteristics of a PAYG funding mechanism are presented.

The pension system we consider only provides retirement pensions, while survivor benefits, invalidity benefits and withdrawals are not included. The population is divided into four different states to which each individual may belong: active (1), pensioner (2), dead (3), unemployed (4). The transition probabilities between states depend on age and time. This implies that the eligibility for pension benefits is independent from the years of service. We define the transition probability of an individual aged x in state i at time t to arrive in state j at time $t+h$ as ${}_hp^{ij}(x,t)$, and the probability for the same individual to remain in state i for time h as ${}_hp^{ii}(x,t)$.

Population dynamics. Let denote $N^i(x,t)$ the number of individuals in state i at age x at time t and $Z^i(x,t)$ the new entrants. The demographic dynamics at each time t is represented as follows:

$$N^i(x,t) = N^i(x-1,t-1)p^{ii}(x-1,t-1) + Z^i(x,t) \quad i = 1,2,3,4 \tag{2.1}$$

The total population in the state i at time t is given by $N^i(t) = \sum_x N^i(x,t)$. The population evolution illustrated in equation 2.1 indicates that the population at time t depends both on the previous-year population who have survived by time t and the new entrants in state i aged x in year t, $Z^i(x,t)$.

The number of new pensioners aged x in year t can be obtained as $Z^2(x,t) = N^1(x-1,t-1)p^{12}(x-1,t-1)$, where the probability $p^{12}(x-1,t-1)$ is zero before the minimum retirement age and one at the maximum retirement age by considering the rules of the system and the retirement propensity.

The number of new deaths aged x occurring in the year t is given by:

$$Z^3(x,t) = N^1(x-1,t-1)p^{13}(x-1,t-1) + N^2(x-1,t-1)p^{23}(x-1,t-1) \tag{2.2}$$

According to the prevalent literature (see, for instance, Gronchi and Nisticò [7] and Alonso-García et al. [1]), the future evolution of individuals in the active state is obtained by the growth rate of the total active population, $\rho(t)$, which equally affects all the contributors, given the initial number $N^1(0)$ of total active population:

$$N^1(t) = N^1(t-1)(1+\rho(t)) \tag{2.3}$$

From the evolution of the active population depends the total number of new actives (at time t), which is calculated as follows:

$$Z^1(t) = N^1(t) - \sum_x N^1(x-1,t-1)p^{11}(x-1,t-1) \tag{2.4}$$

Finally, the future evolution of individuals in the unemployed state is function of the unemployment rate, $\upsilon(t)$, as described in the following equation:

$$N^4(t) = \frac{\upsilon(t)}{1-\upsilon(t)} \cdot N^1(t) \tag{2.5}$$

Similarly to the case of the active population, the evolution of the new unemployed population is modeled by:

$$Z^4(t) = N^4(t) - \sum_x N^4(x-1,t-1)p^{44}(x-1,t-1) \tag{2.6}$$

The number of new entrants in state $i = 1,4$ aged x in year t, is calculated by considering the relative age distributions as follows:

$$Z^i(x,t) = Z^i(t)d_z^i(x,t) \quad i = 1,4 \tag{2.7}$$

Where $d_z^i(x,t)$ is the relative age distribution of the new entrants in state i, and $Z^i(t)$ is the total new entrants in state i at time t. For the number of new entrants in the unemployed state, $Z^4(x,t)$, we assume that the new unemployed population has the same relative age distribution of the new actives, that is: $d_z^4(x,t) = d_z^1(x,t)$.

Contributions and benefits. Let denote $s(x,t,i)$ as the wage for the i-th active aged x at time t, assuming the same wage for all the individuals belonging to the same generation, $s(x,t,i) = s(x,t)$ for all i, independently from their past service duration. The individual wage evolves according to a given growth rate, as follows:

$$s(x,t) = s(x,t-1)\,[1+\xi(t)] \tag{2.8}$$

Where $\xi(t)$ is the growth rate of individual wage from $t-1$ to t. The individual wage multiplied by the number of actives for the same age x and time t provides the total wage earned by the active population aged x at time t:

$$S(x,t) = N^1(x,t)s(x,t) \tag{2.9}$$

Denoting $S(t) = \sum_x S(x,t)$ as the total wage at time t, the corresponding average wage is given by:

$$s(t) = \frac{S(t)}{N^1(t)} \tag{2.10}$$

Denoting $c(t)$ as the contribution rate of the pension system at time t, the individual contribution paid by an active aged x at time t is:

$$c(x,t) = c(t)s(x,t) \tag{2.11}$$

While the total contribution for the active population aged x at time t is given by:

$$C(x,t) = N^1(x,t)c(x,t) = c(t)S(x,t) \tag{2.12}$$

Finally, the amount of total contribution earned by the system at time t is:

$$C(t) = \sum_x C(x,t) = c(t)S(t) \tag{2.13}$$

A NDC system is characterized by a "defined-contribution" design, therefore the contribution rate is fixed and assumed constant over time, $c(t) = c$ for all t. The contributions of each participant are noted on individual notional accounts and are remunerated each year t at a common rate of return, $g(t)$. Differently from financial defined contribution scheme, contributions are not invested in financial market and $g(t)$ is a notional rate established in the design in order to assure the financial stability of the system[1]. The individual notional account for an active i aged x at the end of year t evolves as follows:

$$m(x,t,i) = [m(x-1,t-1,i) + c(t)s(x,t)]\,[1+g(t)] \tag{2.14}$$

For an unemployed i aged x the individual account at the end of year t only changes for the rate of return as there are no new contributions paid:

$$m(x,t,i) = m(x-1,t-1,i)\,[1+g(t)] \tag{2.15}$$

At retirement, the initial benefit is determined by converting the individual notional account into an annuity consistently with the remaining cohort life expectancy, the expected indexation rate, λ^*, and the expected rate of return, g^*. The annuity rate at time t for a new pensioner aged x, $\ddot{a}(x,t)$, assuming that benefits are paid in advance, is determined as:

$$\ddot{a}(x,t) = \sum_{h=0} {}_hp^{22*}(x,t) \cdot \prod_{k=t}^{t+h} \left[\frac{1+\lambda^*(k)}{1+g^*(k)}\right] \tag{2.16}$$

where ${}_hp^{22*}(x,t)$ is the expected survival probability of a pensioner aged x at time t for h years. Eq. 2.16 can be rewritten as a function of a "deviation rate", $j^*(k) = \frac{1+g^*(k)}{1+\lambda^*(k)} - 1$, measuring the amount by which the notional rate deviates from pension indexation (see Gronchi and Nisticò [7]), as follows:

$$\ddot{a}(x,t) = \sum_{h=0} {}_hp^{22*}(x,t) \cdot \prod_{k=t}^{t+h} [1+j^*(k)]^{-1} \tag{2.17}$$

Therefore, the initial benefit for a new pensioner aged x in the year t is calculated as:

$$b_z(x,t,i) = \frac{m(x,t,i)}{\ddot{a}(x,t)} \tag{2.18}$$

While, the total benefits paid to the new retirees in the year t are given by:

$$B_z(x,t) = \sum_{i \in Z^2(x,t)} b_z(x,t,i) \tag{2.19}$$

[1] Countries that introduced NDC schemes chose different rates: e.g., an average of the GDP growth rate in Italy, the per capita growth rate of the contribution payment in Sweden. As observed by Holzmann [8]: "In an economic and demographic steady-state environment, the key variables all offer the same value for the implicit rate of return of an unfunded scheme: the growth rate of the labor force plus the rate of productivity growth."

The total pensions paid to all retirees aged x in the year t evolve as follows:

$$B(x,t) = B(x-1,t-1)p^{22}(x-1,t-1)\,[1+\lambda(t-1)] + B_z(x,t) \tag{2.20}$$

Note that the pension indexation experienced by the pensioners, λ, could be different from its estimated value, λ^*.
Denoting $B(t) = \sum_x B(x,t)$ the amount of total pensions paid to retirees in the year t, the corresponding average pension is given by:

$$b(t) = \frac{B(t)}{N^2(t)} \tag{2.21}$$

Financial sustainability and social adequacy of a PAYG pension system. In a balanced PAYG scheme income from contributions are equal to expenditure on pensions, $C(t) = B(t)$. Combining equations 2.10 and 2.13 in the left side, and remembering equation 2.21 for the right, we obtain the following equilibrium equation:

$$N^1(t) \cdot c(t) \cdot s(t) = N^2(t) \cdot b(t) \tag{2.22}$$

We define $\hat{c}(t)$ as the contribution rate satisfying the equilibrium equation:

$$\hat{c}(t) = \frac{N^2(t)}{N^1(t)} \cdot \frac{b(t)}{s(t)} \tag{2.23}$$

where the ratio $\frac{N^2(t)}{N^1(t)}$, that measures the proportion of the pensioners over active population, is usually called dependency ratio and will be denoted as $D(t)$ in the following. The ratio $\frac{b(t)}{s(t)}$ is the average replacement rate of the system in the year t and will be denoted as $r(t)$. Consequently, the equilibrium equation can be rewritten as:

$$\hat{c}(t) = D(t) \cdot r(t) \tag{2.24}$$

A PAYG system could experience periods with cash-flow deficit (surplus), $C(t) < B(t)$ ($C(t) > B(t)$). We denote as unfunded liabilities (UL) the difference between the pension expenditures and the income from contributions:

$$UL(t) = B(t) - C(t) \tag{2.25}$$

Even if a PAYG scheme is a non-funded system, a buffer or reserve fund could be introduced in order to handle unexpected demographic and economic shocks. The reserve fund emerges from the difference between income and expenditure, so it increases when the unfunded liabilities are negative ($UL < 0$) and decreases when $UL > 0$. The evolution of the reserve fund $F(t)$ in $t \in [0,T]$, with $F(t) \gtreqless 0$, is given by:

$$F(t) = F(t-1)\,[1+g(t-1)] + C(t) - B(t) = F(t-1)\,[1+g(t-1)] - UL(t) \tag{2.26}$$

Where the rate of return of the reserve fund is assumed equal to the notional rate, g. We assume no reserve fund at initial time, $F(0) = 0$.
As observed by Boado-Penas et al. [2], the annual cash-flow deficit/surplus is often considered as a solvency indicator of the pension system, but UL is only a liquidity indicator. In order to measure the financial sustainability of the system, an actuarial balance should be compiled. In PAYG system (NDC or DB), actuarial balance is usually compiled by comparing the Net Present Value of pension expenditures and income from contributions in a long time horizon, $NPV(0,T)$, defined as follows (see Godinez-Olivares at al. [6]):

$$NPV(0,T) = \sum_{t=1}^{T} B(t) \prod_{h=0}^{t-1} [1+g(h)]^{-1} - \sum_{t=1}^{T} C(t) \prod_{h=0}^{t-1} [1+g(h)]^{-1} \tag{2.27}$$

Equation 2.27 can be rewritten as in term of present value of the future unfunded liabilities $UL(t)$, $t = 1,2,...,T$:

$$NPV(0,T) = -\sum_{t=1}^{T} UL(t) \prod_{h=0}^{t-1} [1+g(h)]^{-1} \tag{2.28}$$

A pension system will be sustainable if the NPV over a long time horizon is not negative, $NPV(0,T) \geq 0$.
If we introduce the reserve fund, the financial sustainability of the system can be measured by its value in each year: a pension system will be sustainable if $F(t) \geq 0 \; \forall t$ over a long time horizon.
It is interesting to note that the sustainability in terms of NPV can be expressed in terms of reserve fund. Starting from equation 2.26, the expression of the reserve fund at time T is given by:

$$F(T) = F(0) \prod_{h=0}^{t-1} [1+g(h)] - \sum_{t=1}^{T} UL(t) \prod_{h=t}^{T-1} [1+g(h)] \tag{2.29}$$

dividing by $\prod_{h=0}^{t-1} [1+g(h)]$ we obtain:

$$F(T) \prod_{h=0}^{T-1} [1+g(h)]^{-1} - F(0) = NPV(0,T) \tag{2.30}$$

therefore, assuming $F(0) = 0$, $NPV(0,T) \geq 0$ is equivalent to $F(T) \geq 0$.
In our numerical application we will study the system's financial sustainability through the analysis of the reserve fund in each year $t \in [0,T]$.
Demographic evolution and/or economic dynamics could undermine the PAYG equilibrium. While in a pure Defined Benefit-PAYG system, being the benefits fixed, the equilibrium can be restored through a change of the contribution rate, in a NDC system, where the contribution rate should be constant, the equilibrium is obtained by changing (usually reducing) the replacement rate. This is automatically obtained through changes in the notional rate, strictly linked to the economic conditions, and changes in the expected probabilities used in equation 2.16, reflecting life-expectancy evolution[2].
Changes in replacement rate could allow the system to be financially sustainable, but may produce inadequate pension benefits, reducing the living standard of pensioners and making the system not attractive for new entrants. Therefore a "social sustainability" issue could arise. As observed by Schokkaert [18], "an equitable and credible promise should relate future pensions to the future average living standard in society", consequently social sustainability could be measured comparing the average pension paid by the system and the average wage earned by the active population. Following Devolder et al. [4], we use the replacement rate, $r(t)$, to represent the social adequacy of a NDC system.

3 Modeling demographic and macroeconomic variables

In order to study the evolution of a NDC pension system under a pandemic crisis and its financial and social sustainability, assumption on transition probabilities and macroeconomic variables evolution should be done. We introduce in our model two sources of risk: demographic (longevity) risk and economic risk. Specifically, we assume that all the transition probabilities between the four states considered are deterministic, except for death probabilities for pensioners, $p^{23}(x,t)$. Moreover, we model through time series process the three macroeconomic variables that directly affect the pension system: unemployment rate $\upsilon(t)$, wage growth rate $\xi(t)$ and inflation rate $i(t)$.
First, we choose the best models for the four variables, fitting them on data referred to the time period immediately preceding the crisis, then we apply a deterministic shock in the first year of projection whose amount is based on information about the impact of COVID-19 on mortality and economics known at

[2] As observed by Devolder et al. [4], there are situations where a NDC system is not able to immediately restore the equilibrium, therefore it remains vulnerable to demographic and economic shocks

the time of writing. We assume independence between demographic and economic variables. Details on the model selection process are provided in the following, a full description of the dataset used and fitting results are provided in the next section.

Lee-Carter model and ARIMA With reference to pensioners' mortality, we assume a Poisson distribution for the number of deaths: $D(x,t) \sim Poisson(E(x,t)m(x,t))$, where $E(x,t)$ and $m(x,t)$ are the exposed to risk of death and the central death rate for age x and year t, respectively. We adopt a Lee-Carter model, which is considered a benchmark in the literature on mortality modeling, describing the central death rates of pensioners by the following equation:

$$\log m(x,t) = \alpha_x + \beta_x \kappa_t \tag{3.1}$$

where α_x is the static age function, β_x is the non-parametric age-period term and κ_t is the mortality time index. The corresponding death probabilities $p(x,t)$ are derived from:

$$p(x,t) = 1 - \exp(-m(x,t)) \tag{3.2}$$

The mortality forecast is obtained by modeling and forecasting the time index κ_t by an Auto-Regressive Integrated Moving Average (ARIMA) process with a Gaussian white noise with mean 0 and variance σ_κ. The ARIMA(p,d,q) model, where p is the order of the AR model, d is the degree of differencing and q is the order of the MA model, has the form:

$$(1 - \sum_{k=1}^{p} \alpha_k L^k)(1-L)^d X_t = (1 + \sum_{i=1}^{q} \beta_k L^k)\varepsilon_t \tag{3.3}$$

Where L is the lag operator.

VAR and VECM The macroeconomic variables are modeled as multivariate time series: we chose the best model among Vector Auto-Regressive (VAR) and Vector Error Correction Term (VECM) classes of models for inflation rate and wage growth rate, and chose the best model within the ARIMA class of models for the unemployment rate. Unemployment rate is modeled apart due to the different nature of this variable with respect to inflation rate and wage growth rate. Indeed, inflation rate and wage growth rate represent variations from one year to the next, while the unemployment rate is a ratio between specific groups of people.
A VAR(p) model has the form:

$$Y_t = A_1 Y_{t-1} + \ldots + A_p Y_{t-p} + U_t \tag{3.4}$$

where $Y_t = (Y_{1t}, \ldots, Y_{kt}, \ldots, Y_{Kt})$ is a set of variables, A_j are $(K \times K)$ coefficient matrices for $j = 1, \ldots, p$ and u_t is a K-dimensional white-noise process with $E(u_t) = 0$ and $E(u_t u_t^\top) = \Sigma_u$.
A VECM (in the transitory specification) model has the form:

$$\Delta Y_t = \alpha\beta^\top Y_{t-1} + \Gamma_1 \Delta Y_{t-1} + \ldots + \Gamma_{p-1} Y_{t-p+1} + U_t \tag{3.5}$$

with

$$\Gamma_j = -(A_{j+1} + \ldots + A_p) \quad \text{for } j = 1, \ldots, p-1$$
$$\alpha\beta^\top = -(I - A_1 - \ldots - A_p)$$

where α is the loading matrix and β contains the long-run relationship coefficients. These classes of models are the most widespread in the macroeconomic literature (see, for example, Robinson [17]; Lack [11]; Ette et al. [5]; Power and Gasser [15]).

Each macroeconomic variable is forecasted using a mean-reversion to an exogenous long-run trend, similarly to the approach in Lee and Tuljapurkar [12][3], where the main reason to include an exogenous long-run trend is to control the long-run simulations. Indeed, statistical time-series usually are not planned for long-term projections, and without this constrain they could provide unrealistic values. Thus, we will choose the best ARIMA-CM (Constrained Mean) for the unemployment rate and the best either a VAR-CM or a VECM-CM model for inflation rate and wage growth rate. This approach meets our scope, which is to obtain reliable forecasts and simulation in line with the expected long-term trends and give a robust stochastic structure to our framework in order to study the impact of COVID-19 on the NDC pension systems.

As the unemployment rate affects the growth rate of the active population, the latter will be defined as a function of the former and an exogenous long-run trend, τ, according to the approach suggested by Lee and Tuljapurkar [12]:

$$1+\rho(t)=\frac{1+\tau-\upsilon(t)}{1+\tau-\upsilon(t-1)} \tag{3.6}$$

Note that, if the unemployment rate is constant, then $\rho=0$, i.e. the active population remains stable, while if it increases (decreases), ρ decreases (increases).

We organize the model choice procedure as follows. For all the macroeconomic variables, we firstly check the stationarity of the time series through the Phillips-Perron (PP) test and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test, both at a 5% significance level. The first one tests the null hypothesis of non-stationarity (i.e. to have a unit root) against a stationarity alternative, while the KPSS tests the null hypothesis of stationarity against a non-stationarity alternative. Then, in order to choose the best ARIMA-CM model for the unemployment rate, we analyze the plots of Auto-Correlation and Partial Auto-Correlation functions, the Akaike Information Criterium (AIC) and the residuals (by testing the stationarity, the non auto-correlation and the normality hypothesis). Regarding the wage growth rate and the inflation rate, we perform the Granger casuality test, we search the optimum number of parameters for the VAR model and test the validity of the model similarly to what we do for the unemployment rate.

4 Numerical application

The numerical application is focused on the analysis of the impact of COVID-19 on the Italian NDC pension system. The NDC system has been introduced in Italy by the Dini reform in 1995. It is based on notional accounts, which are fed by contributions valorized with a rate of return related to nominal GDP growth (as a five-year moving average). At retirement, the accumulated notional capital is converted into an annuity taking into account average life expectancy at retirement. Currently, despite the introduction of the NDC, the system is far from being balanced essentially because of the continuous increase of the elderly population and the decrease in the number of active workers paying contributions. These conditions could jeopardize the future adequacy of the pension benefits.

Essentially, it is likely that the COVID-19 pandemic affects the demographic and economic conditions of the pension system in the form of a shift in the main demographic and economic variables involved in the system. Therefore, in order to estimate the pandemic effects on the NDC system, we apply a shock to the employment rate, wage growth rate, inflation rate and death rate. Specifically, we assume a reduction in the employment, inflation and wage growth rates, and an increase in the death rate.

The numerical analysis has been developed over 75 years, which is a very long time horizon, but consistent with most of the actuarial reports assessing the pension systems' financial sustainability (e.g.

[3] The authors observed that the structural changes that happened in recent years in some key variables of a social security system, such as fertility, productivity, and interest rate, resulted in mean values that differ from the average of their past values. After much experimentation, they found that satisfactory forecasts were obtained by pre-specifying the long-term means of the series, rather than estimating them from the data.

US and Canada use 75 years, while Japan 95 years). A 75-year time horizon allows for exhausting at the end of the projections the baby-boomers cohort, thus almost eliminating its effect on the final demographic structure. This time horizon is used in the last long-term projections of Italian pension expenditure developed by the Ministry of Finance is 2018-2070 (Ragioneria Generale dello Stato [16]). The economic data consist of 37 past observations over the period 1983-2019 (the mortality data until 2017).

The reference population used in the analysis has been built from the demographic and economic structure of the National Employees' Pension Fund members. The assumptions concerning the demographic and economic evolution have been taken from the long-term projections of Italian pension expenditure developed by the Ministry of Finance (Ragioneria Generale dello Stato [16]).

We consider an initial active population $N^1(0)$ composed of 1,000 males with an age distribution carried from the employees' observed distribution in the Italian National Institute of Social Security (INPS) pension scheme in 2015. The initial wage distribution of the active population by age derives from the employees' observed wage distribution in the INPS pension scheme in 2015. The initial number of pensioners is set according to the dependency ratio of the INPS pension scheme for employees in 2015, i.e. $N^2(0) = N^1(0) \cdot D(0)$ with $D(0) = 43.6\%$. The initial age distribution of both pensioners and pension benefits derives from the corresponding observed distribution of the INPS pension scheme in 2015. The age distribution of new actives, $d_z^1(x,t)$, is taken from the observed age distribution of actives with past service duration less than 2 years in 2015. The same holds for the age distribution of the new unemployed population, $d_z^4(x,t)$, that, by assumption, it is equal to the age distribution of new actives. Finally, we assumed that $d_z^1(x,t)$ is constant over time.

With regards to the transition probabilities from active to pensioner, $p^{12}(x,t)$, we assume that all the actives retire at age 63, therefore $p^{12}(x,t) = 0 \; \forall x < 63$ and $p^{12}(x,t) = 1 \; \forall x \geq 63$. We make the same assumption for the unemployed ($p^{42}(x,t) = 0 \; \forall x < 63$ and $p^{42}(x,t) = 1 \; \forall x \geq 63$. Age 63 has been chosen consistently with the average retirement age of Italian employees in 2015.

In our analysis, we do not consider the active population mortality because of the features of the Italian system, which disregards the distribution of inheritance gains from people who die before the earliest possible retirement age. Therefore, the notional accounts are not affected by the active population mortality, thus $p^{13}(x,t) = 0$ for all ages and time. The same assumption is done for the unemployed, $p^{43}(x,t) = 0 \; \forall x,t$. With respect to pensioners' death probabilities, we assume that they are equal to the corresponding probabilities for the Italian general population. Both deaths $D(x,t)$ and exposures to risk $E(x,t)$ involved in the calculation of the pensioners' probability of death refer to years 1983-2015 and are taken from the Human Mortality Database. The probabilities for the remaining years, 2016-2019, are obtained from the Lee-Carter model best estimate.

The contribution rate $c(t)$ is set to 30% according to the Italian pension system, which has fixed it to 33% but with the inclusion of the invalidity benefits, survivors' benefits, and withdrawals. These latter benefits are not considered in our case study.

The data on the macroeconomic variables are taken from the Italian National Institute of Statistics (ISTAT). Specifically, υ is taken from the unemployment rate of the male population aged 25-75 over the period 1983-2015, for the remaining years (2016-2019), it is obtained by regression analysis from the unemployment rate of the male population aged 15-64. $\xi(t)$, which is a nominal rate, derives from the wage growth rate for the period 1983-2015, and from the gross contractual hourly remuneration of employees for the last four years[4]. No adjustment is made for inflation rate, $i(t)$.

The impact of the COVID-19 pandemic crisis on an NDC system is addressed by introducing a shock in $t = 2020$ on the unemployment rate, $\upsilon(t)$, the wage growth rate, $\xi(t)$, the inflation rate, $i(t)$, and the mortality probabilities, $p^{23}(x,t)$. The shock levels for the wage growth rate and the inflation rate are set according to the estimates provided by ISTAT, regarding the estimation of the COVID-19 impact on the Italian economy in 2020 (ISTAT [10]). The shock level for the unemployment rate is set according

[4]The two quantities differ less than 10^{-4} in the last 5 years of jointly data

to the estimates provided by OECD for Italy (OECD [13]). The mortality shock is set in order to obtain the pandemic extra deaths (36,002) occurred until October 5, 2020.

The analysis is organized in three steps. The first one consists of choosing the best model for the macroeconomic variables (unemployment rate, inflation rate, wage growth rate) among the approaches proposed in the previous section, and to estimate the parameters of the Lee-Carter model used to describe the mortality of pensioners. The second step regards the stochastic projections of the variables characterizing the NDC system with and without the COVID-19 crisis. To do this, we simulate 1000 trajectories. The final step consists of analyzing the impact of the COVID-19 scenario on the reserve fund and replacement rate of the pension system over the 75-year time horizon.

Regarding the macroeconomic time series, we firstly check their stationarity through the PP test (null hypothesis: no stationarity) and the KPSS test (null hypothesis: stationarity) both at a 5% level. We find the wage growth rate and inflation rate as stationary; hence, we cannot proceed by studying the cointegration of the series and consider the VECM models that require first-differencing of the time series. On the contrary, for the unemployment rate, we cannot reject the null hypothesis of both tests, making it difficult to state if it is stationary or not. It is worth remembering that these tests perform asymptotically and that in a finite sample is very hard to distinguish between a trend-stationary and a difference-stationary behavior. An inappropriate transformation could cause serious issues in the forecasting. Therefore, we decided to not differencing the series and considering the ARMA models instead of the ARIMA.

Concerning the unemployment rate, $\upsilon(t)$, the Auto-Correlation function (ACF) and the Partial Auto-Correlation functions (PACF) plots suggest to analyze the models with both AR and MA components until lag 5. We consider ARMA models with AR component always present, at least at lag 1 (since pure MA models forecasting give exactly the mean of the process after a period equal to the chosen lag period, and it is not what we are looking for). We eliminate the models with convergence problems and the ones having no-significant parameters for at least the greatest lag of AR or MA component, then the choice reduces to one model among ARMA(1,1), AR(2), and AR (4). Hence, we analyze the AIC values and perform the log-likelihood ratio test at a 5% significance level, finding AR(4) as the best model. Finally, we check the validity of the model through a residual analysis, confirming that residuals are stationary (we reject the PP test at a 5% level) and not auto-correlated (no significative lags emerge from the ACF and PACF plots, and the null hypothesis of the Ljung Box Test is rejected for all lags at a 5% level).

Table 4.1: ARMA model selection

	ARMA(1,1)	AR(4)	AR (2)
AIC	51.087	50.442	55.624
LLR p-value		0.0101*	

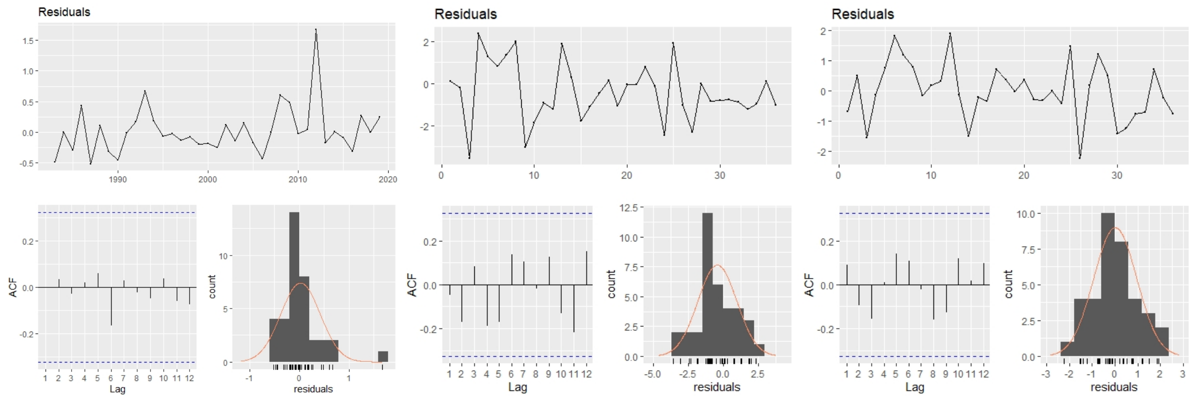
**The comparison can only be done between AR(4) and AR(2).*

The residuals distribution, the ACF and PACF are provided in Fig. 4.1 for the unemployment rate (panel a), the wage growth rate (panel b) and the inflation rate (panel c). While, Fig. 4.2 illustrate the Ljung-Box Q test.

Table 4.2: AR(4). z test of coefficients.

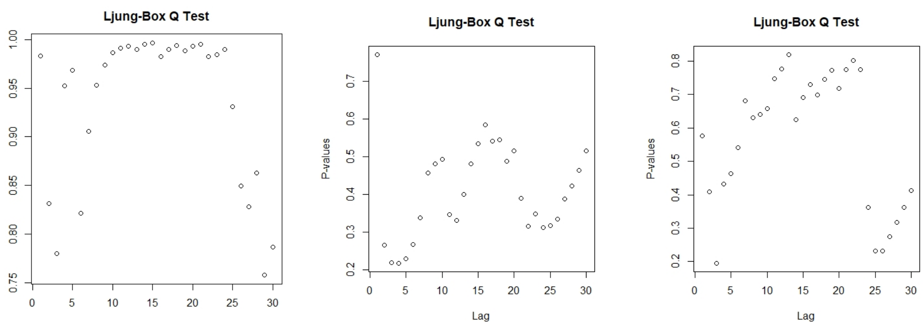
	Estimate	Std. Error	z value	Pr(> $|z|$)
AR(1)	1.87451	0.14670	12.7777	$< 2.2e-16$***
AR(2)	-1.51386	0.30281	-4.9994	$5.751e^{-07}$***
AR(3)	0.99334	0.30114	3.2986	0.0009716***
AR(4)	-0.41998	0.15102	-2.7809	0.0054213**

Signif. codes: *p<0.05, **p<0.01, ***p<0.001.



(a) Unemployment rate (b) Wage growth rate (c) Inflation rate

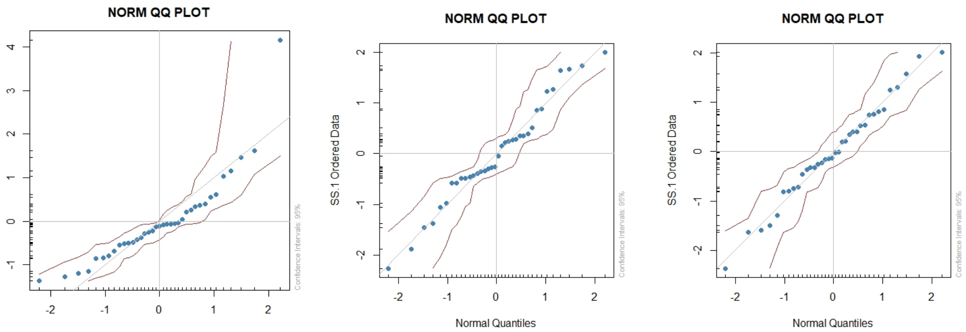
Figure 4.1: Residuals distribution, ACF and PACF for unemployment rate, wage growth rate and inflation rate.



(a) Unemployment rate (b) Wage growth rate (c) Inflation rate

Figure 4.2: Ljung-Box Q test for unemployment rate, wage growth rate and inflation rate.

We graphically check if the distribution of the residuals is Gaussian (the mean is really close to 0 and there is only 1 observation over the 3σ interval) also considering the QQ plot (see Fig. 4.3) that gives satisfactory results. The final model for the unemployment rate is AR(4) with a 5.5% exogenous long-run trend.



(a) Unemployment rate (b) Wage growth rate (c) Inflation rate

Figure 4.3: QQ plot for unemployment rate, wage growth rate and inflation rate.

For the inflation rate and wage growth rate time series, that are stationary, we perform the Granger casuality test at a 10% level. The results of the test show that inflation rate can help to predict wage growth rate at lag 1, and that wage growth rate can help to predict inflation rate at lag 2. Therefore, modeling them together can help to improve the forecast and it is sensible to continue the study with VAR models. Therefore, we continue the analysis using the VAR models instead of studying them separately. We find VAR(1) the best model. Similarly to the approach followed unemployment rate, we then check for both wage growth rate and inflation rate, the stationarity of residuals (PP test is rejected at a 5% level), their non auto-correlations (no significative lags emerge from the ACF and PACF plots, and the null hypothesis of the Ljung Box Test is rejected for all lags at a 5% level), their normality distributions (the mean is really close to 0 and there are no observations over the 3σ interval. They graphically seem to approximately distribute like a Gaussian and the QQ plots confirm this intuition (Fig. 4.3). The final model for the inflation rate and wage growth rate is VAR(1), with an exogenous long-run trend of 4% for the wage growth rate and 1.5% for the inflation rate.

The long-term macroeconomic assumptions are coherent with the standard ones of the Ministry of Finance for the projection of the national pension expenditure (Ragioneria Generale dello Stato [16]). Therefore, we assume that the GDP growth rate is equal to the sum of the growth rate of the active population $\rho(t)$, growth rate of labour productivity and inflation rate $i(t)$. Moreover, the growth rate of the individual wage, $\xi(t)$ and growth rate of the labour productivity are assumed equal.

According to the features of the Italian NDC system, the notional rate, $g(t)$, is chosen equal to the GDP growth rate, and the pension indexation rate, $\lambda(t)$, is equal to the inflation rate $i(t)$.

4.1 Results

Fig. 4.4 for the baseline scenario and Fig. 4.5 for the COVID-19 scenario illustrate the dynamics of dependency ratio, replacement rate, equilibrium contribution rate, average wage[5], average pension[6], and the ratio of contributions to pensions.

[5] Net of inflation. Logarithmic scale.

[6] Net of inflation. Logarithmic scale.

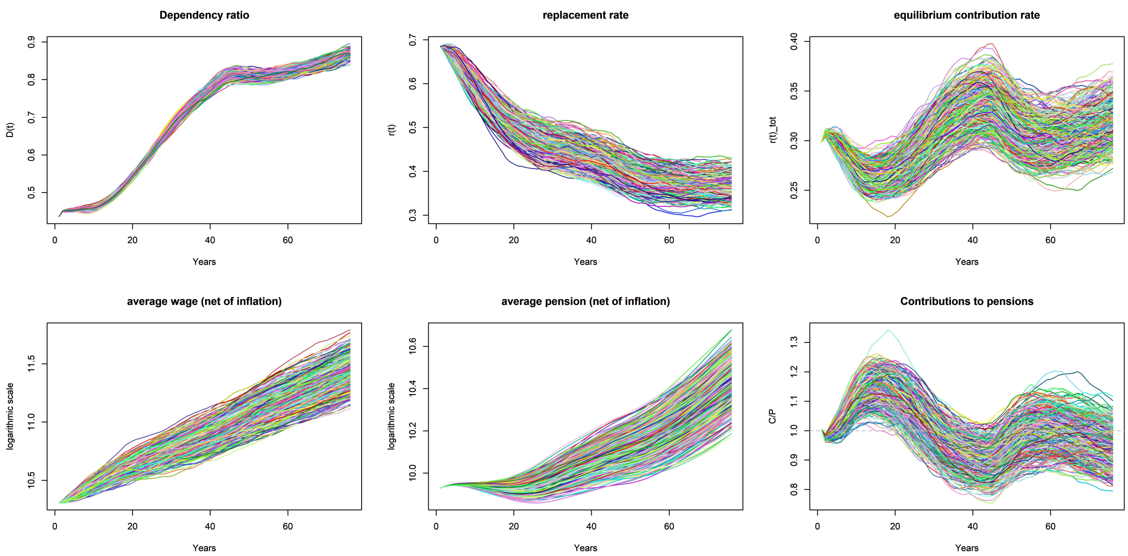


Figure 4.4: Pension system evolution in the base scenario. Years 2019-2094.

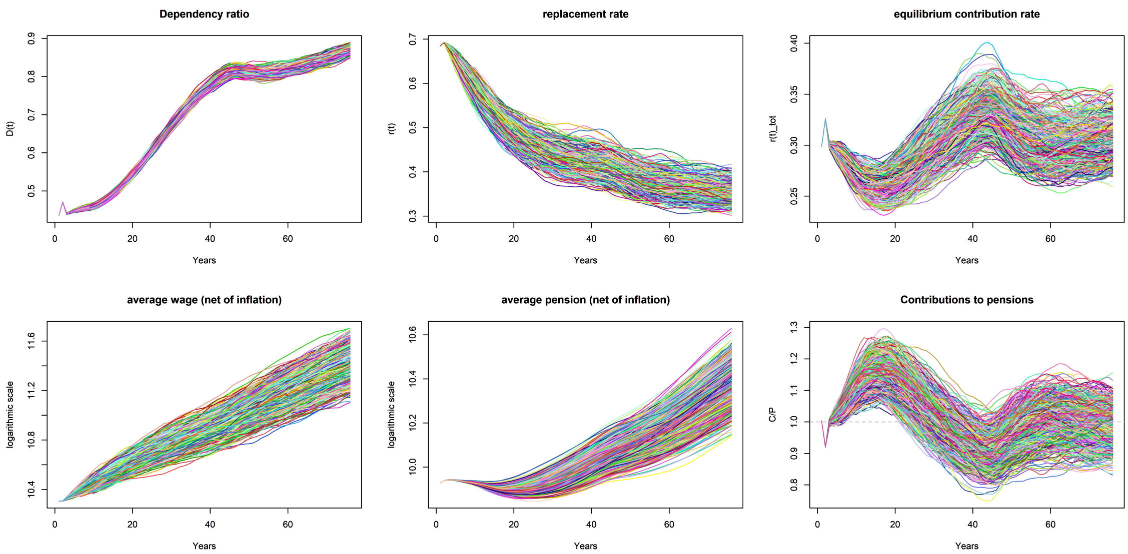


Figure 4.5: Pension system evolution in the COVID-19 scenario. Years 2019-2094.

The dependency ratio increases from 43.6% to 86.8% in 75 years in both scenarios, showing a very small impact of the COVID-19 pandemic in the medium/long run. Nevertheless, the number of pensioners per worker doubles during the time horizon, due to the life expectancy increase, which immediately results in a worsening of the financial sustainability of the pension system. As expected in the short-run, we observe a more significant impact of the pandemic on the dependency ratio due to the sudden increase of the unemployment rate: in 2020, the COVID-19 shock on average caused a 2% increase of the dependency ratio, which fall to -0.5% after 5 years and becomes +0.3% after 15 year[7]. The dependency ratio is affected in opposite directions by the mortality shock (that reduces the pensioners and influences the system in the next years) and the unemployment shock (that reduces the working population, and whose effect is absorbed after a few years). Therefore, we observe a low variability of this indicator

[7] Respectively: 47.1% vs 45.1%; 45.2% vs 45.6%; 50.3% vs 50%.

across the forecast horizon[8].

The COVID-19 shock has an immediate impact on the average wage[9] that persists over time. The shock impacts about -1.3% in 2020, -0.6% in the medium term (in 2054), and -0.7% in 2094. As for the average pension (at current prices), the impact of COVID-19 is more visible in the medium and in the long run. In fact, the shock affects the notional account of active workers, that will be converted into a pension benefit in the following years, in two ways: firstly, some workers will lose their jobs due to the rising unemployment rate[10], secondly and more important, GDP becomes negative involving a negative notional rate for active people in 2020. The average COVID-19 impact on pensions is -1.1% after 5 years, -3.5% after 35 years, and -4.2% at the end of the time horizon.

The replacement rate[11], which gives a measure of social adequacy, decreases from 68.5% to 36.7% in the baseline case and 35.4% in the COVID-19 scenario. These outcomes reveal a social adequacy issue in the Italian NDC system in the medium-long period: after only 15 years the reduction in the baseline (shock) case is approximately -15.6% (-17%). In the COVID-19 scenario, the replacement rate in 2020 is higher than in the baseline case, since the pandemic shock has an immediate impact on the average wage and not on the average pension. In the medium term (after 15-45 years) the average pension is more affected by the pandemic: for example, the replacement rate in the COVID-19 scenario is about -1.4% after 15 years, and -1.3% after 35 years.

In the baseline scenario, the equilibrium contribution rate (the ratio between total pensions and total wages) fluctuates around 30% during the entire time horizon[12], decreasing in the medium term and increasing in the long one. This ratio does not reach unsustainable values over the time horizon and, in the end, it is in line with the initial fixed contribution rate. The COVID-19 shock has an immediate adverse impact by increasing the equilibrium contribution rate (+1.8% in 2020) but has a favorable impact in the following years (-1.1% in 2094). As previously observed, the pandemic shock does not influence the dependency ratio but reduces the replacement rate.

As a liquidity indicator, we evaluate the ratio between contributions income and pensions expenditure, $\frac{C(t)}{P(t)}$. If $\frac{C(t)}{P(t)} = 1$, the PAYG pension system is in equilibrium. If $\frac{C(t)}{P(t)} > 1$, the fund $F(t)$ is increased. The opposite is true when $\frac{C(t)}{P(t)} < 1$. In our analysis, the average ratio stays in the range (90%,116%) during the whole time horizon for both scenarios. It shows a significant variability in the medium/long term, i.e., its standard deviation in both scenarios exceed 4% for $t = 15$ and the 99% confidence interval, at the end of the time horizon, is (82.8%, 107.4%) for the baseline scenario and (85.4%,111%) in the COVID-19 one. The pandemic shock results in an immediate reduction of the ratio in 2020 (-5.4%), in an increase in the medium term (+2.2% in $t = 15$) and in the long term (+3.4% in $t = 75$). The overall effect (joint with the timing of the deficits/surpluses) can be evaluated by analyzing the reserve fund value, $F(t)$, at the end of the time horizon.

Without the shock, the system is not financially sustainable, providing a final fund of about -63.5 million euro[13]. However, in the COVID-19 scenario, the final fund has a positive value of about 137 million euro, that means the system is financially sustainable over the 75-year time horizon. Therefore, the shock has improved the financial sustainability of the system. This result may be surprising, but it is in line with previous considerations. Actually, the main effect of the pandemic shock is on the notional account, therefore on future pensions. As a consequence, the impact of the COVID-19 on the pension system turns into a social adequacy problem, since, in the long-run, pensions suffer heavier reduction

[8]For example, in $t = 75$ the standard deviation is approximately 0.71% (0.73%) in the baseline (pandemic shock) scenario, while the 99% confidence interval of $D(t)$ is (85%,88.6%) for both scenarios.

[9]Before the payment of contributions and considering current prices.

[10]Consequently, they will not pay contributions and, at the end of their working life, will have a smaller notional account to convert into a pension.

[11]It is worth remembering that this is an average replacement rate of the entire system and not the ratio between the first pension and the last wage of a new pensioner.

[12]In fact, it is given by the product of dependency ratio and replacement rate that move in opposite directions.

[13]Note that the total contributions (in the baseline scenario) are about 90 million euro in 2020 and 170 million euro in 2094.

with respect to salaries, than in the baseline case. This result is critical if we consider that even in the baseline case there is a problem of social adequacy.

5 Conclusions

This paper focused on the impact of the COVID-19 pandemic on the financial sustainability and the adequacy of pension benefits to retired of the Italian NDC pension system. Indeed, the restrictive measures on economic activity introduced to counter the spread of the virus are affecting the labor market by reducing employment and wages, thus probably lowering contributions income. Furthermore, the rate of return on notional accounts, that in Italy is an average of the GDP growth rate, will also be affected by the pandemic, and future pensions will be accordingly reduced. Finally, the pandemic is producing an increase in mortality rates, mostly at older ages, reducing the benefits paid to pensioners.

In order to address the effects of the COVID-19 on the pension system, the macroeconomic variables involved in calculating contributions and pensions (unemployment rate, wage growth rate and inflation rate), and mortality rates are modeled as stochastic time series. We introduce the impact of the pandemic as a deterministic shock.

The outcomes show that, in the long run, there is no strong impact on the dependency ratio, while an immediate reduction of about 1% on the average wages is observed, which remains flat for the entire time horizon. A greater reduction (3%-5%) on the average pension affects future new pensioners who experienced the COVID-19 during their working life. The motivation lays in the main impact of the pandemic on the unemployment rate and mostly on the GDP, which is used as the notional rate of the NDC system. As a consequence, the social adequacy of the pension system is worsened with respect to the shock-free scenario that already exhibited replacement rate critical values. However, the financial sustainability of the system does not suffer from the pandemic shock since the ratio of contributions to pensions increases up to 3.5% on average. The financial sustainability of the NDC system is confirmed by the reserve fund at the end of the time horizon that shows a higher value under the COVID-19 scenario than in the baseline one. In conclusion, in the long run, the Italian system seems to be resilient to the pandemic shock but, at the same time, it shows a critical evolution of social adequacy.

Future research will focus on the introduction of a floor in the reduction of the notional rate in case of a pandemic shock, that may distribute the COVID-19 consequences between social adequacy and financial sustainability of the pension system. Moreover, we will study the introduction in the system of automatic balancing mechanisms aiming at reducing financial sustainability and social adequacy issues.

References

[1] Alonso-García, J., Boado-Penas, M. d. C., Devolder, P. (2018). Automatic balancing mechanisms for notional defined contribution accounts in the presence of uncertainty. Scandinavian Actuarial Journal **2**, 85–108.

[2] Boado-Penas, M. d. C., Valdés-Prieto, S., Vidal-Meliá, C. (2008). The Actuarial Balance Sheet for Pay-As-You-Go Finance: Solvency Indicators for Spain and Sweden. Fiscal Studies, **29**(1): 89–134.

[3] Chłoń-Domińczak, A., Franco, D., Palmer, E. (2012). The first wave of NDC reforms: The experiences of Italy, Latvia, Poland and Sweden. In R. Holzmann, E. Palmer and D. Robalino. NDC pension schemes in a changing pension world. Volume 1: Progress, lessons, and implementation. Washington, DC, World Bank.

[4] Devolder, P., Levantesi, S., Menzietti, M. (accepted). Automatic Balance Mechanisms for Notional Defined Contribution pension systems guaranteeing social adequacy and financial sustainability. Annals of Operations Research.

[5] Ette, H., Uchendu, B. Uyodhu, V., (2012). Arima Fit to Nigerian Unemployment Data. Journal of Basic and Applied Scientific Research,**2**(6): 5964–5970

[6] Godinez-Olivares, H., Boado-Penas, M. C., Haberman, S. (2016). Optimal strategies for pay-as-you-go finance: a sustainability framework. Insurance: Mathematics and Economics, **69**: 117–126.

[7] Gronchi, S., Nisticò, S. (2006). Implementing the NDC theoretical model: A comparison of Italy and Sweden, in Holzmann, R., Palmer, G. (Eds.), Pension reform: Issues and prospect for non-financial defined contribution (NDC) schemes, chapter 19, 493-515, Washington, DC, World Bank.

[8] Holzmann, R. (2017). The ABCs of nonfinancial defined contribution (NDC) schemes. International Social Security Review **70** (3), 53–77.

[9] IMF (2020). Pension schemes in the COVID-19 crisis: Impacts and policy considerations. Special Series on COVID-19. International Monetary Fund, Fiscal Affairs.

[10] Istituto Nazionale di Statistica (ISTAT) (2020). *Le prospettive per l'economia italiana nel 2020-2021*. Rome.

[11] Lack, C. (2006). Forecasting Swiss inflation using VAR models. Swiss National Bank Economic Studies, 2.

[12] Lee, R., Tuljapurkar, S., (1998). Stochastic forecasts for social security. Frontiers in the Economics of Aging. National Bureau of Economic Research: 393-428.

[13] OECD (2020)*Employment Outlook OECD 2020*, OECD Publishing, Paris, July 7, 2020.

[14] Palmer, E. (2006). What's ndc?. In: Holzmann, R., Palmer, E. (Eds.), Pension Reform: Issues and Prospects for Non-Financial Defined Contribution (NDC) Schemes. The World Bank, Washington, D.C.: 17–35, (Chapter 2).

[15] Power, B. Gasser, K. (2012). Forecasting Future Unemployment Rates. ECON 452 First Report.

[16] Ragioneria Generale dello Stato (RGS) (2018). *Le tendenze di medio-lungo periodo del sistema pensionistico e socio-sanitario - Aggiornamento 2018*. Report, 19, Rome.

[17] Robinson, W. (1998). Forecasting inflation using VAR analysis. Bank of Jamaica.

[18] Schokkaert, E., Devolder, P., Hindriks, J., Vandenbroucke, F. (2018). Towards an equitable and sustainable points system. A proposal for pension reform in Belgium. Journal of Pension Economics and Finance, 1–31. doi:10.1017/S1474747218000112

[19] WHO. Virtual press conference on COVID-19—11 March 2020. March 11, 2020. `https://www.who.int/docs/default-source/coronaviruse/transcripts/who-audioemergencies-coronavirus-press-conference-fulland-final-11mar2020.pdf?sfvrsn=cb432bb3_2` (accessed March 20, 2020).