Pricing and hedging in incomplete markets by maximum entropy and convex duality.

By

Prof. Jose Luis Vilar-Zanón (Dept of Financial and Actuarial Economy & Statistics, Universidad Complutense de Madrid)

Prof. Barbara Rogo (Dept of Statistical Science, Sapienza University of Rome)

Summary: In this seminar we deal with the most important problems of Financial Mathematics, valuing financial claims and hedging their risk. These topics are important issues from an actuarial point of view because financial options are embedded in insurance products (like the participating life insurance policies), making their pricing, risk assessment, and management processes key elements of the insurance product suitability.

On the Finance side, we will engage with important tools like the *Fundamental Pricing Theorem* (FPT) of *Arbitrage Pricing Theory* (APT), making the important distinction between *natural and risk neutral probabilities*. We will introduce the concept of *incomplete market* and see its consequence regarding the FPT formulation. We will also work with the concept of *efficient prices* finding an access to them through the *entropy maximization* of the risk neutral probability measure. This entails the application of *Entropy Pricing Theory* (EPT) and more generally, the use of an *inverse method* whose advantage consists of guaranteeing its functioning with any stochastic model of the market scenarios we might choose. When the risk hedging comes up, market incompleteness makes impossible the resolution through a replicating portfolio of the claim payoff. We must work in a set of *pseudo-replicating portfolios*, each one implying a *remaining risk* (not hedged by the portfolio) and an *excess* representing gains, both from the claim issuer point of view (the insurance company). We then apply *risk measures* to calculate an optimal pseudo-replicating portfolio. These risk measures are the *conditional value at risk* (CVaR), a coherent risk measure, and its associated *value at risk* (VaR) which is the risk measure applied by *the Solvency II European Directive*. Also following this Solvency II Directive, the pricing step must conform to the *market consistency principle,* so it can be qualified either as *mark-to market* or *mark-to-model*.

All this journey is completed thanks to the application of sound mathematical programming methods. These are *convex optimization* to calculate the *no arbitrage prices* of the claim, *linear programming* to calculate the CVaR-optimal pseudo replicating portfolio and its associated VaR, and *Fenchel Duality Theory* to show how the hedging step can be deduced by duality from any of the pricing primal mathematical programs.

We exemplify by means of a *participating life insurance policy* embedding a *cliquet guarantee*. We use the *Heston model* that includes volatility risk for the underlying dynamic, showing how it can be calibrated from *market data* by means of the Nelder and Mead *derivative free optimization algorithm*. We show how to calculate either the no arbitrage prices of the cliquet guarantee according to the market consistency principle (mark-to-model), and their associated optimal static hedging positions, and finally show how to assess and manage its risk.