

Persistent bugs and misrepresentation in Italy's regional electoral laws

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Abstract

Electoral laws sometimes produce contradictory results. This is the case in many countries and also in procedures adopted for the election of 19 Italian regional councils. The current Italian regional laws use a variety combinations and adjustments to traditional proportional formulae in the effort to find a balance between governability and representation. This survey questions whether such complex and inextricable procedures are really necessary or just the result of compromise and historical stratification, which are making them basically incomprehensible to voters. It also shows how a flaw in achieving double proportionality in the national law has been passed onto the regional laws. Even in those few cases the legislator has acknowledged the bi-proportional nature of the problem, attempts to solve it remain unsatisfactory.

Keywords: survey of electoral laws, Italian regions, bi-proportional apportionment, optimization approach

1. Introduction

Soon after the reform of the Constitution in 2001¹, in the midst of a lively debate on electoral systems, Italian Regions started working to issue their own electoral legislation. The constitutional reform modified the organization of the Italian Republic and the so-called “Title V”, clarified the relationship between different levels of government within the Nation (central, regional, provincial and municipal) and their respective powers. Some Italian Regions with a “special Statute”² already exercised a certain degree of autonomy, especially as concerns financial issues, and already had their own electoral laws; but for all other Regions, the rules and procedures for electing councils were set nationally.

The first regional council elections were held in 1970 according to a unique and fully proportional electoral system. When Italy’s Parliament passed to a mixed system this was reflected on regional councils too with a reform passed in 1995 that established a 20% majority and 80% proportional formula³. Today⁴, consistently with the constitutional reform, each Region can adopt its own electoral rule. National guidelines only require that a regional Statute be established first and that fundamental principles as concerns eligibility and ineligibility of candidates and the length of each term (five years) be respected. The Statute specifies the council size and organization of the Region.

By now, most Regions have adopted specific legislation or modified some aspects of the previous electoral procedures. Out of the 13 Italian Regions that held political elections in March 2010, six (Campania, Lazio, Marche, Friuli Venezia Giulia, Puglia, Toscana) did so with new electoral laws; for the remaining, the election was still ruled by the frame established in 1968 and modified in 1995⁵.

Despite the differences, current legislation for electing regional councils share two main features:

- the quest for a balance between governability and representation;
- the idea that representation should guarantee proportionality of the council seats with respect to both the votes cast to the parties and population in each constituency.

In this paper we discuss the old and new Italian regional electoral laws, describing the details and analyzing the different characteristics and their consequences on the electoral result. We question whether the complex procedures adopted are really necessary or just

¹ Italian Constitutional Law 3/2001.

² The Italian Regions with special statute are: Sicilia, Sardegna, Valle D’Aosta, Friuli Venezia Giulia and the two autonomous Provinces of Trento and Bolzano.

³ However, in the elections for national Parliament the share of majority and proportional is the inverse, with 75% of the seat assigned through a first-past-the-post.

⁴ Law 165/2004, "Disposizioni di attuazione dell’articolo 122, primo comma, della Costituzione"

⁵ Italian Law 108/1968 and 43/1995 (“Tatarella” law). As long as Regions do not subscribe regional statutes, the election of the regional council is ruled by the last electoral law in charge. Moreover, among those which passed a regional statute, Emilia Romagna, Liguria, Piemonte e Umbria did not change the electoral procedure.

the result of compromise and historical stratification, which are making the laws basically incomprehensible to voters. We also focus on the problems that may arise when these laws are applied in practice. We show that the proportional principle underlying these laws does not meet expectations and the council size fixed in the regional Statutes is rarely respected. An exception is found in the laws of Marche and Friuli Venezia Giulia which take this problem into account, but do not effectively solve it.

2. The basic mechanism underlying the regional seat allocation procedures

The basic mechanism for most regional elections is still regulated by the law n. 108/1968 and its modifications, also known as the “Tatarella law”⁶. According to this law - and its adjustments introduced from 1975 to 1995⁷ - the size of the regional council is equal to 80, 60, 50, 40 or 30 seats depending on whether the size of the population registered in the most recent census is respectively greater than 6 million, between 4 and 6 million, between 3 and 4 million, between 1 and 3 million, and less than 1 million inhabitants.

The regional council is elected through a mixed system: 80 percent of the representatives with votes cast at a district level (districts corresponding to the provinces of the Region) on the basis of a proportional rule, while the remaining 20 percent is elected at large through a general ticket.

The number of representatives to be elected in each district is fixed on the basis of the most recent population census by applying the Hare/Largest Remainders method⁸. District lists are open, with the possibility for the voter to indicate a preferred candidate. On the other hand, the regional lists for the general ticket are blocked, i.e., the order of the candidates in the list is pre-established and no preferences required. The regional lists each support a candidate to Governor or President of the regional council.

A *group* is a set of lists in different districts identified by the same symbol. Each district list or group must support a regional list. Single lists and groups supporting the same regional list form a *coalition*. In Figure 1, list A in district 1 and 3 and list B in district 1 and 2 identify two different groups; while the group of list A and the single list C form a coalition supporting the candidate Governor in regional list X, the group of list B and the single list D form a coalition supporting the candidate Governor in list Y.

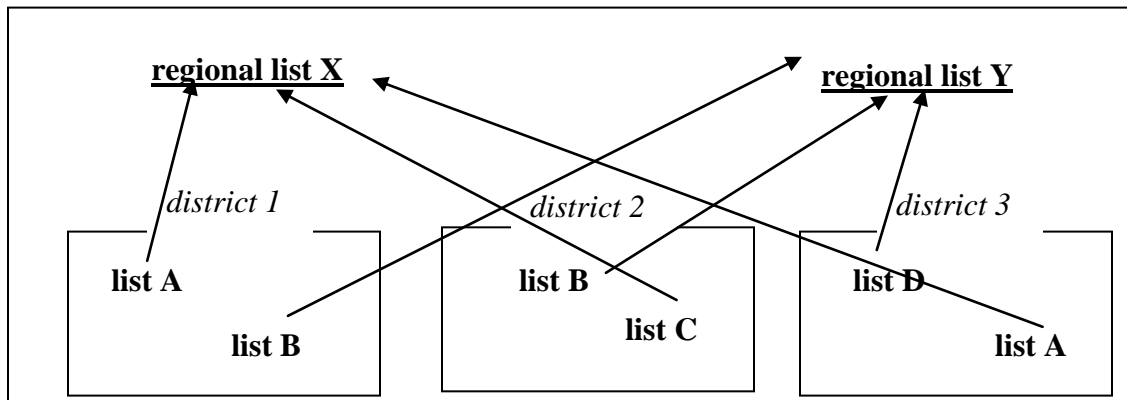
There is a single ballot, but disjoint vote is allowed, that is, one can vote for, say, a district list A and simultaneously for a regional list B (and Governor Y) that is not supported by A.

⁶ Law n. 43/1995, called “Tatarella” from the name of its promoter, is formally a modification of the previous fully proportional electoral law n. 108/ 1968 rather than a new law.

⁷ L. n. 39/1975; L. n. 130/1975; L. n. 154/1981; L. n. 53/1990; L. n. 271/1991; L. n. 16/1992; L. n. 30/1994; L. n. 43/1995.

⁸ For a general overview of seat allocation procedures see e.g. Lakeman and Lambert (1955), Balinski and Young (1982), Grilli di Cortona et al. (1999).

Figure 1 – Groups and coalitions of district lists



This double-layered structure suggests that a major concern of the regional electoral systems is to guarantee appropriate territorial representation and governability within the council at the same time. Indeed, district lists can stand for local peculiarities and allow for a closer relation between the candidates and the voters, while regional lists assure allegiance to a specific Governor, whatever her/his district-level supporters. This purpose is confirmed and reinforced by the choice of a mixed system: proportional for the district level and plurality for the regional level.

The actual procedure to transform votes into seats requires a long series of computations performed by district and regional offices. Different formulae are used along the way and it may even happen that different procedures can be used to perform the same step. This certainly contributes to some confusion on which concept of proportionality is being pursued (Pennisi, 1999; Grill di Cortona et al, 1999). The following paragraphs detail the district and regional level steps.

2.1 Proportional representation at the district level

With an aim to reduce party fragmentation, exclusion thresholds keep district lists with relatively small support out of the contest. In order to participate to the seat assignment a single list or group must obtain at least 3% of the total vote or must be connected to a regional list which obtained at least 5% of the total vote.

Proportional representation is pursued using a Largest Remainders logic applied in two stages: first district by district, then to the subset of remaining seats which are dealt with at the regional level by combining the single district residuals. The actual formula used varies according to the stage (district level or regional level) and it can also vary according to the district.

In fact, at the district level, the Hagenbach-Bischoff quota is applied first, by calculating the ratio between the number of votes in district i and the number of seats in district i plus one ($q_i = \lfloor v_i / (s_i + 1) \rfloor$). The number of seats assigned to each list within a district corresponds to the votes gained divided by the quota and rounded down ($s^{ij} = \lfloor v_{ij} / q_i \rfloor$). However, should the assignment of Hagenbach-Bischoff quotas produce a total number

of seats exceeding the actual number of seats at stake in a given district⁹, the Hare quota (simple ratio between votes and seats ($q_i = \lfloor v_i/s_i \rfloor$), is used in that district instead. The district quota basically represents the price of a seat in terms of votes. The procedure, therefore, allows for this price to be fixed according to different rules (Hagenbach-Bischoff or Hare) in each district and the rule chosen depends on the configuration of the vote.

After assigning seats according to Hagenbach-Bischoff or Hare quotas in each district, some seats may still be left empty. Hence they are grouped together at regional level ($s^i = s_i - \sum_j s^{ij}$). The number of *residual votes*, corresponding to those votes of the district lists that did not contribute to the first seat gains ($r_{ij} = v_{ij} - s^{ij} \cdot q_i$) are also set aside and handled at regional level, after summing up residual votes coming from all district lists belonging to the same group¹⁰. These seats and residual votes form a sub-problem to which the Hare quota and Largest Remainders method is applied. Here the Hare quota is given by the ratio of the total residual votes and leftover seats ($\sum_{ij} r_{ij} / \sum_i s^i$) rounded down and determines the number of seats assigned to each single list or group on the basis of residual votes. Finally, the remaining seats originating from the Hare quotas are assigned to groups on the basis of the largest fractional remainders and, in case of a tie, the actual number of residual votes is used as a tie-break.

At this point all the available seats have been divided among groups/lists, but they must be now assigned to each list in a specific district. For each district, lists are ordered on the basis of their residual votes scaled by the district quota (i.e., r_{ij}/q_i)¹¹. The seats are assigned to single lists in districts depending on their ranking of scaled residuals. In other words, the seats obtained by a given group are allocated to the district with the greatest value of scaled residual among those in which the group is represented. Note that the scaled residual $r_{ij}/q_i = v_{ij}/q_i - s^{ij}$ is the same as the fractional part of the ratio already applied to each list at the district level. So, basically, the seats are redistributed to each group on the basis of their largest remainders calculated in each district (but the assignment resulting from the procedure is different from the one obtained by assigning remaining seats with largest remainders district by district).

Following this procedure the ranking may assign seats in districts which are already saturated. In fact, the redistribution of seats at provincial level usually changes the original apportionment by allowing for additional seats. That this might occur is straightforward given that even districts for which all seats were assigned during the first phase contribute with their residual votes to the second phase. The issue here is that provincial representation sought by the law can be substantially altered. Table 1 shows how this actually happened, for example, in the 2010 elections in Lombardy, where in six districts over twelve the number of seats assigned by the proportional procedure ended up being greater (or smaller) than the amount at stake.

⁹ For example, should $\lfloor v_{ij}/q_i \rfloor = v_{ij}/q_i$ the ratio already being an integer for all lists in district i , the total number of seats assigned (obtained by summing over j) would turn out to be equal to $s_i + 1$.

¹⁰ Districts which have already completed the seat allocation during the first phase of the procedure also contribute with their residual votes.

¹¹ Note that the district quota may have been defined either with Hagenbach-Bischoff or with Hare.

Table 1 – Proportional seat assignment at district level in Lombardy in 2010 elections

districts	seats to be allocated	seats assigned with full quota	remaining seats to be assigned	total seats assigned with proportional rule	difference
Bergamo	7	6	1	7	0
Brescia	8	5	4	9	1
Como	4	2	2	4	0
Cremona	2	0	2	2	0
Lecco	2	0	3	3	1
Lodi	1	0	0	0	-1
Mantova	3	2	1	3	0
Milano	21	19	4	23	2
Monza e Brianza	5	4	0	4	-1
Pavia	4	3	0	3	-1
Sondrio	1	0	1	1	0
Varese	6	5	0	5	-1
Total	64	46	18	64	0

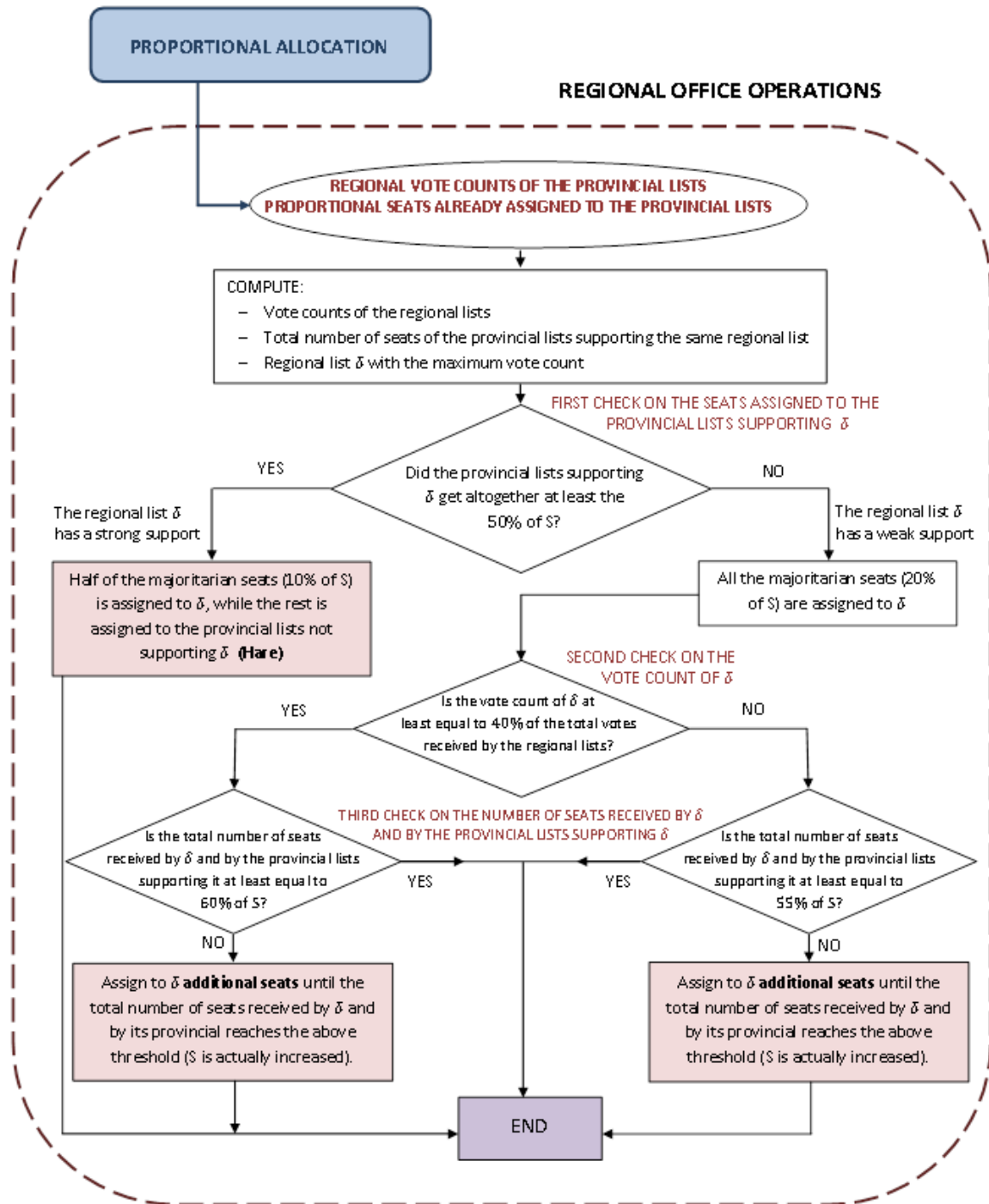
2.2 Plurality at the regional level

The regional lists are usually grouped into coalitions supporting a candidate Governor. The coalition (or single list) that receives a plurality of votes elects the Governor/President of the Region. In order to ensure a large majority in the council for the Governor's coalition, a prize accounting for 10% to 20% of the total number of seats is foreseen. This majority prize determines the actual number of candidates being elected in the regional list.

Of course, the support the Governor can gather in the council depends on the amount of seats the district lists belonging to his/her coalition have already gained. If the Governor's coalition has won altogether at least 50% of the total seats in the council with the proportional rule, there already is a strong support. In this case, only half of the regional seats (equivalent to 10% of the overall number of seats) are filled with the regional candidates accompanying the Governor, while the rest is assigned to the provincial lists not supporting him/her. These seats are distributed among such lists with a proportional rule, the Hare quota plus largest remainders applied to the residual votes following the procedure already carried out at the district level as illustrated in paragraph 2.1, skipping districts to which scaled residuals have been previously assigned.

On the other hand, if the Governor's coalition wins less than 50%, additional support is needed to ensure governability. In this case all regional seats (20% of the total) are filled by candidates from the regional winning list. This count may, however, not be sufficient: despite the 20% prize the Governor might still not hold a majority in the council. So the procedure carries out two further checks. If the votes cast for the Governor's coalition in the general ticket are at least equal to 40% of the total votes received by the regional lists, but the total number of seats awarded up to now (including his/her district supporters) does not reach 60%, special seats are added to the council until the mentioned threshold is reached. Should the votes cast for the Governor's coalition be less than 40%, special seats are added to the council up to a 55% threshold (see Figure 2, where δ is the Governor's list or coalition). In both cases the council size is increased.

Figure 2 – The seat assignment and majority prize at the regional level according to Law n. 108/1968 and further modifications



The special seats are taken by candidates in the district lists supporting the Governor and not from the winning regional list. They are assigned by a proportional Hare quota and Largest Remainders method applied to the residual votes, following the procedure

already carried out at the district level as illustrated in Section 2.1, skipping districts to which scaled residuals have been previously assigned¹².

In conclusion, the procedure followed for the election of regional councils is inherently complex and convoluted: it requires several electoral formulae at the same time, it depends on several different parameters and thresholds and the successive computations involve alternatively the district and regional level. Moreover, territorial seat allocation can be altered twice. First during the proportional representation stage, by assigning seats to districts where they were no longer available; and then during the plurality stage, where special seats may inflate the council size in order to guarantee governability (see Table 2).

Table 2 – Total seat assignment in Lombardy at district level in 2010 elections

districts	seats to be allocated	seats assigned with proportional rule	seats assigned during the plurality stage	total	difference
Bergamo	7	7	2	9	2
Brescia	8	9	1	10	2
Como	4	4	0	4	0
Cremona	2	2	0	2	0
Lecco	2	3	0	3	1
Lodi	1	0	1	1	0
Mantova	3	3	0	3	0
Milano	21	23	0	23	2
Monza e Brianza	5	4	1	5	0
Pavia	4	3	0	3	-1
Sondrio	1	1	1	2	1
Varese	6	5	1	6	0
Total	64	64	7	71	7

3. Variations on the same theme and empirical evidence from the 2010 elections

Some Regions still use the “Tatarella” formula, some use slightly modified the rules and some have actually switched from a mixed system to a fully proportional one (see Table 4). Whether Regions have or not adopted their own electoral legislation, most have nonetheless increased in their Statute the size of the regional council with respect to the “Tatarella law”. As of 2010, Basilicata and Veneto were the only regions still having not adopted a Statute and, thus, the number of representatives in their councils is still the one established by the 1968 law (i.e., 30 and 60 for Basilicata and Veneto, respectively). In the following, we disregard the two autonomous Provinces of Trento and Bolzano (which also have their own electoral system).

Table 3 shows how council sizes have inflated since the introduction of regional electoral laws to the current legislation. Council size includes the President/Governor and we

¹² If the all scaled residuals in the ranking have already been used to assign additional seats, the procedure starts from the first district all over again.

calculate the average price per seat in terms of the ratio between the number of residents and the number of seats. Data refer to the 2001 population census. By comparing the columns, one can appreciate not only the increase in the council size most Regions opted for in establishing a proper electoral law, but how this implied a reduction of up to 50% in the theoretical number of votes needed to obtain a seat. The disparity in the price of a seat per region already present under “Tatarella” is even wider under the current arrangements.

Table 3 – Size of the Italian Regional Councils under the previous and the 2010 legislation

Region	council size		residents per seat		(b)/(a)
	“Tatarella” law	current legislation	price of a seat “Tatarella” (a)	price of a seat in the current law (b)	
Piemonte *	60	60	70244.6	70244.6	1.00
Valle D'Aosta *	30	35	3984.9	3415.7	0.86
Lombardia	80	80	112906.9	112906.9	1.00
Veneto	60	60	75461.6	75461.6	1.00
Friuli Venezia Giulia *	40	59	29594.1	20063.8	0.68
Liguria	40	51	39294.6	30819.3	0.78
Emilia Romagna	50	50	79666.9	79666.9	1.00
Toscana *	50	55	69956.1	63596.5	0.91
Umbria *	30	31	27527.5	26639.5	0.97
Marche *	40	43	36764.5	34199.6	0.93
Lazio *	60	71	85206.9	72005.8	0.85
Abruzzo *	40	42	31559.8	30057.0	0.95
Molise	30	30	10686.7	10686.7	1.00
Campania *	60	61	95032.2	93474.3	0.98
Puglia *	60	70	67011.8	57438.7	0.86
Basilicata *	30	30	19925.6	19925.6	1.00
Calabria *	40	50	50286.7	40229.3	0.80
Sicilia *	60	90	82816.5	55211.0	0.67
Sardegna *	40	80	40797.0	20398.5	0.50

* Regions which had issued by March 2010 their own electoral law.

Note: The last population census, used to calculate (a) and (b), was in 2001. In this table and the following, we disregard the two autonomous Provinces of Trento and Bolzano, which also have their own electoral system. The council size includes the President/Governor.

Indeed, the size of current regional councils has recently been put under scrutiny by citizens and the central Government, also in response to a new wave of corruption scandals. Since decree-law 138/2011¹³ decreasing substantially the number of council members to levels lower than those established in the “Tatarella” law, is one of the parameters used in order to determine financial premiums (or sanctions) related to the transfer of resources from the central level to the regional administrations. Further action announced by the Italian Government in 2012 is to modify the 2001 Constitutional

¹³ Article 14 provides an incentive to Regions to adopt the following upper bounds to the regional council size: 80, 70, 50, 40, 30 or 20 seats depending on whether the size of the population registered in the most recent census is respectively greater than 8 million, between 6 and 8 million, between 4 and 6 million, between 2 and 4 million, between 1 and 2 million, and less than 1 million inhabitants.

reform in the direction of further clarifying responsibilities of the various layers of government and reducing the devolution of legislative power on such issues.

A summary of the electoral laws in force in the Italian Regions is given in Table 4.

A novel issue in many regional laws is the introduction of gender quotas to encourage female candidates. In 8 regions out of 19 the number of candidates of a same gender cannot exceed the two thirds of the total number of seats at stake in the corresponding constituency (when fractional, this number is rounded to the closest integer). In others, the quota is relaxed to 60 percent or strengthened to 70 percent, while in Valle D'Aosta the threshold is established by below, in the sense that the percentage of candidates of each gender must be at least 20 percent. Calabria seems to be the less concerned with this issue, requiring just one candidate for each gender to be present in each district list.

In some cases the 20 percent plurality system is abandoned in favor of a fully proportional electoral system with a generous majority prize. This holds for Campania, Sicilia, Marche, Friuli Venezia Giulia and Toscana which switch to the d'Hondt divisor formula. Valle D'Aosta is a special matter because here, if the majority prize implies a strong correction, instead of granting it, a second vote is called for the two biggest lists. However, in most cases, only small variants to the "Tatarella" law were actually introduced in the new electoral laws. Regions have basically modified exclusion thresholds, introduced minority warrants or considered different types of list. Although most regions still adopt a vote threshold for district lists (or groups) of 3 per cent of the total vote or requiring to be connected to a regional list which obtained at least 5 per cent of the total vote, for example, Calabria and Puglia have simplified the requirement for district lists (groups) to simply count at least 4 per cent. Moreover, Umbria, Campania, and Toscana have all inserted specific clauses to ensure that, despite the winner's majority prize, the losing coalition or lists get at least 35 per cent of the seats; in Friuli Venezia Giulia, this percentage is risen to 40 per cent.

All regional laws - except Tuscany's - require seats to be allocated to districts before the elections on the basis of the most recent population census and according to the Hare quota /Largest Remainders method¹⁴.

¹⁴ No inconsistencies in totals arise in Tuscany given that the number of seats per constituency is not predetermined by the law. In any event, as concerns the 2010 elections Tuscany is one of the few Regions for which seats assigned match those that would have been allocated to constituencies according to Hare quota/largest remainder and the latest population census.

Table 4 – Main features of the Italian Regional Councils’ elections

	Region	General features				Regional level seat assignment				District level seat assignment	
		Law	Most recent election year	Type of electoral system	Gender quota	Exclusion thresholds	Electoral formula 1	Majority prize	Minority warrant	Exclusion thresholds	Electoral formula 2
"Tatarellum law" still adopted	Lombardia	Law 108/1968 and modifications in Law 43/1995	2010	mixed (20% M and 80% PR)	no	no	20% Plurality	yes	no	yes	80% PR Hagenbach-Bischoff quota or PR Hare quota (if necessary) + Largest Remainders within the region
	Veneto		2010								
	Liguria		2010								
	Sardegna		2009								
	Emilia Romagna Molise		2010 2011								
Slightly adjusted "Tatarella law"	Piemonte	LR. 21/2009	2010	mixed (20% M and 80% PR)	at most 2/3 same gender	no	20% Plurality	yes	no	yes	80% PR Hagenbach-Bischoff quota or PR Hare quota (if necessary) + Largest Remainders within the region
	Lazio	LR. 2/2005	2010		at most 2/3 same gender						
	Abruzzo	LR. 9/2005	2008		at most 70% same gender						
	Puglia	LR. 2/2005	2010		at most 2/3 same gender						
	Basilicata	LR. 3/2010	2010		no						
	Calabria	LR 1/2005	2010		at least one candidate of each gender						
Other	Umbria	LR 2/2010	2010	mixed (20% M and 80% PR)	at most 2/3 same gender	no	20% Plurality	yes	yes	yes	80% PR Hagenbach-Bischoff quota or PR Hare quota (if necessary) + Largest remainders within each
	Valle D'Aosta	LR 3/1993	2008	100% PR (and possible double)	at least 20% of each	yes	PR-Hare (and possible)	yes	no	does not apply	
	Campania	LR. 4/2009	2010	100% PR	at most 2/3 same gender	yes	PR-d'Hondt	yes	yes	no	LR-Hagenbach-Bischoff + Largest remainders within the region
	Sicilia	LR 7/2005	2012	100% PR	at most 2/3 same gender	yes	PR-Hare	yes	no	no	LR-Hare + Largest remainders within each district
	Marche	LR. 27/2004	2010	100% PR	at most 2/3 same gender	yes	PR-d'Hondt	yes	no	no	PR-Hagenbach-Bischoff and Largest remainders taking into account the number of seats at
	Friuli Venezia Giulia	LR. 17/2007	2008		at most 60% same gender	yes	PR-d'Hondt	yes	yes	no	PR-Imperiali quota and Largest remainders taking into account the number of seats at
	Toscana	LR. 25/2004	2010		at most 2/3 same gender	yes	PR-d'Hondt	yes	yes	does not apply	

Legend: M = majority system; PR = proportional system

Therefore, put aside minor details, most regional elections are still performed according to the procedure ruled by the 1968 law and its modifications. This means that a combination of proportional formulae, complex majority prizes and possible inflation of the council size are still foreseen. Pitfalls of the original law are inherited by the new ones¹⁵.

This holds in particular for inconsistencies in territorial representation. The Italian regional electoral laws are not fit to handle the double dimensionality of the problem they must solve, which require seats to be simultaneously proportional to both district size and list/group totals. As observed elsewhere, this feature is not new and the procedure used to elect the Italian Chamber of Deputies contains the same flaw (Pennisi, 2006; Pennisi et al., 2006, 2007). The debate in recent years may have created some awareness on this issue given that two Regions - Marche and Friuli Venezia Giulia - have explicitly intended to address it when establishing their current electoral system, although they do not provide a satisfactory solution.

In Marche¹⁶ the number of seats allocated to each district is established on the basis of the latest population census and it is computed according to the Hare/Largest Remainders method. The provincial lists, the groups of lists and the coalitions are defined as in the "Tatarella" law, but the electoral system adopted is a proportional one as in the former 1968 law with a majority prize.

The ballot is single and voters express a preference for a candidate Governor and for one of the lists that support her/him (disjoint vote is not allowed). A prize is assigned to the coalition with the largest number of votes in order to guarantee at least a 25 seat majority in the council. The remaining seats are distributed to the other coalitions with the d'Hondt method and, among each coalition, to the lists or groups of lists on the basis of the Hagenbach-Bischoff quota. The residual votes of each list or group are used to assign the possible remaining seats.

At district level, the distribution of the seats to the lists is performed by the Hare quota, but only seats corresponding to full quotas are assigned, while the remainders are scaled by the total number of votes in the constituency, obtaining a value for each provincial list (called *residual votes*). A global ranking of all the lists is then obtained at a regional level on the basis of such values and it is used to assign the remaining seats. Within the constituencies the electoral procedure checks the total number of seats obtained by each group at a regional level against the one obtained by the same group (as the ensemble of the corresponding provincial lists). If the latter is greater than the former, seats are subtracted to the lists (considering them altogether and following the decreasing order of their seats) until the two amounts match. These seats are added to those left-over after the

¹⁵ Complications and confusion in applying the regional electoral rules also arise in practice. Press reviews at the time of the elections in Calabria report disagreements on the actual winners and a "bug" in the computer system responsible for determining the results. Inconsistencies between the number of seats assigned to the province of Catanzaro and Crotona were brought to court (cfr. <http://ultimaora.wordpress.com/2010/03/31/calabria-ecco-il-consiglio-regionale-2010/>; http://www.calabriaonline.com/articoli/ricorsi-e-polemiche-sull-assegnazione-dei-seggi-nel-consiglio-regionale_692.htm).

previous step, and then they are assigned to the provincial lists on the basis of their global ranking. Following this ordering, seats are assigned to provincial lists belonging to groups that have still not received all their seats. During this step an additional check is performed in order to prevent that the total number of seats allocated to each constituency exceeds the seats at stake.

Although rather complicated, the above procedure guarantees that the prefixed totals for both groups and districts are met by the final seat assignment. Council size inflation and other inconsistencies in territorial representation seem solved. The law for Marche is therefore an attempt to handle the double dimension of proportionality typically sought by the regional electoral systems. However, if on the one hand the totals for groups and constituencies are satisfied, on the other, the allocation of seats to groups within the constituencies is not necessarily the “closest” to a fully proportional one. This is because Marche’s electoral procedure (just like the “Tatarella” law) relies on a global ranking of the list residuals in which all reference, both to the group and the constituency to which a list belongs, are lost¹⁷.

Table 5 – Votes cast and seats assigned at district level in Marche (2010 elections)

VOTES CAST AT DISTRICT LEVEL						
Group/List	ANCONA	ASCOLI PICENO	FERMO	MACERATA	PESARO E URBINO	Total
ALLEANZA PER L'ITALIA	7204	1144	669	4473	1064	14554
ALLEANZA RIFORMISTA	12552	1714	687	1883	2865	19701
ITALIA DEI VALORI	21341	11125	6002	12677	14391	65536
PARTITO DEMOCRATICO	72429	29985	24479	36885	61119	224897
UNIONE DI CENTRO	12238	6399	5000	11568	6784	41989
VERDI	3942	2242	1050	2320	3087	12641
LISTA CIVICA MARCHE	1388	0	747	1658	2481	6274
IL POPOLO DELLA LIBERTA'	61088	29802	27505	51991	55086	225472
LEGA NORD	10963	4424	4289	10205	15845	45726
LADESTRA	2504	1264	1843	2407	1160	9178
INSIEME PER IL PRESIDENTE	1325	5081	423	1857	869	9555
RIFONDAZIONE COMUNISTI ITALIANI	8759	4923	3244	4951	6098	27975
SINISTRA ECOLOGIA LIBERTA'	7322	2771	2693	3151	3171	19108
Total votes	223055	100874	78631	146026	174020	722606
SEAT ASSIGNMENT						
Group/List	ANCONA	ASCOLI PICENO	FERMO	MACERATA	PESARO E URBINO	Total
ALLEANZA PER L'ITALIA	1	0	0	0	0	1
ALLEANZA RIFORMISTA	1	0	0	0	0	1
ITALIA DEI VALORI	1	1	0	1	1	4
PARTITO DEMOCRATICO	4	2	2	3	4	15
UNIONE DI CENTRO	0	1	1	1	0	3
VERDI	1	0	0	0	0	1
LISTA CIVICA MARCHE	0	0	0	0	0	0
IL POPOLO DELLA LIBERTA'	3	2	2	2	3	12
LEGA NORD	0	0	0	1	1	2
LADESTRA	0	0	0	0	0	0

¹⁷ Consider, for example, a given district i and two lists, j and h , each competing in i . Suppose that both j and h already received seats on the basis of the Hare quota. Let r_{ij} and r_{ih} be the residual votes in constituency i for j and h respectively; suppose that we are now assigning a remaining seat and that j in i is the next element to consider in the global ranking, and h in i follows. According to the procedure described above, it may happen that, even if $r_{ij} > r_{ih}$, the seat is assigned to h due to the fact that the group of j has already all its seats, while the group of h does not. In this case, proportionality is not satisfied in district i .

INSIEME PER IL PRESIDENTE	0	0	0	1	0	1
RIFONDAZIONE COMUNISTI ITALIANI	1	0	0	0	0	1
SINISTRA ECOLOGIA LIBERTA'	1	0	0	0	0	1
Total seats	13	6	5	9	9	42

Note: The council size in Marche is 42. The President is not counted in the seat allocation.

This can be seen by analyzing the results of the 2010 elections, where the seats at stake in the five provincial districts were 13, 6, 5, 9 and 9 according to the data of the latest population census. Three coalitions fought in the competition and lists won the seats shown in the last column of Table 6. The sum of the differences between the seats assigned to each list within each district and the exact (albeit fractional) share of seats each list is entitled to according to the Hare quota is 18.75. This is a measure of the deviation between Marche's solution and a fully proportional allocation. We will show that smaller deviations can be obtained for Marche's 2010 elections with alternative seat allocations.

Table 6 – An alternative seat assignment for Marche based on Hare quotas

SEAT ASSIGNMENT "CLOSER" TO EXACT HARE QUOTA						
Group/List	ANCONA	ASCOLI PICENO	FERMO	MACERATA	PESARO E URBINO	Total
ALLEANZA PER L'ITALIA	1	0	0	0	0	1
ALLEANZA RIFORMISTA	1	0	0	0	0	1
ITALIA DEI VALORI	1	1	0	1	1	4
PARTITO DEMOCRATICO	4	2	2	3	4	15
UNIONE DI CENTRO	1	0	1	1	0	3
VERDI	0	1	0	0	0	1
Group/Lista CIVICA MARCHE	0	0	0	0	0	0
IL POPOLO DELLA LIBERTA'	3	1	2	3	3	12
LEGA NORD	0	0	0	1	1	2
LADESTRA	0	0	0	0	0	0
INSIEME PER IL PRESIDENTE	0	1	0	0	0	1
RIFONDAZIONE COMUNISTI ITALIANI	1	0	0	0	0	1
SINISTRA ECOLOGIA LIBERTA'	1	0	0	0	0	1
Total seats	13	6	5	9	9	42

One way to obtain a proportional allocation is to minimize the deviation between a seat allocation and the perfectly proportional one according to a given metric (see, for example, Pennisi, 1999, Pennisi et al., 2007). Developing on this idea in the recent literature, minimizing a given index of proportionality was extended to the case of *biproportional* apportionment. This has led to the formulation of a new class of constrained optimization problems called “best approximation problems” which are solvable in polynomial time through the use of network flow techniques (see, Pukelsheim et al., 2012, Ricca et al., 2012 and the references therein). These models are Mixed Integer Linear Programs (MILP) where the constraints are given by the row and column totals of the seat matrix and by the condition that zero votes must correspond to zero seats. By relaxing the integrality constraint, fractional seat allocations (*target shares*) can

be obtained so as to achieve proportionality at least in theory. In order to restore integrality, the idea is to look for integral apportionments that are as close as possible to the ideal ones in a suitable metric. Different metrics were adopted for the formulation of the objective function to obtain different possible seat allocations. For example, an alternative seat allocation for Marche in 2010 can be produced by actually minimizing the sum of the absolute differences between seats and exact Hare quotas. Indeed, the seat assignment in Table 6 was obtained with this purpose and, as shown in Table 7, the sum of differences between the seats assigned by the alternative MILP method and the exact Hare quotas is smaller than the one corresponding to the 2010 electoral result, proving that the latter is not the optimal solution. Should the deviation from a fully proportional allocation be measured on the basis of the maximum difference, in this case, the MILP solution would still be better.

Table 7 – Measuring deviation from exact Hare quotas for the 2010 Marche’s Electoral result and the solution of the MILP (target shares Hare quotas)

Index	2010 electoral result	MILP solution
max	1.20	0.87
sum	18.75	17.45
average	0.29	0.27

The Hare quota is often looked upon as the ideal fractional allocation for proportional representation. However, applying the Hare quota in each constituency, as do most of the electoral laws illustrated here, does not take into account the double proportional nature of the problem, pertaining both to the rows and columns of the vote per district and per electoral list matrix. Balinski and Demange (1989a and 1989b) introduced the *fair share* as a perfectly proportional allocation in the matrix case, since it is the unique matrix satisfying a list of five proportionality axioms, i.e., *exactness*, *homogeneity*, *uniformity* (or *consistency*), *monotonicity*, and *completeness*. Another possible alternative seat allocation for Marche can then be obtained by the same MILP model considered above, but taking the elements of the fair share matrix as the target quotas. This leads to the seat assignment illustrated in Table 8. Table 9 shows different indices of deviation (average, maximum, sum of single differences) from the fair shares for this seat assignment and for Marche’s 2010 electoral result.

Table 8 – An alternative seat assignment for Marche based on fair shares

SEAT ASSIGNMENT "CLOSER" TO FAIR SHARE						
Group/List	ANCONA	ASCOLI PICENO	FERMO	MACERATA	PESARO E URBINO	Total
ALLEANZA PER L'ITALIA	0	0	1	0	0	1
ALLEANZA RIFORMISTA	0	0	1	0	0	1
ITALIA DEI VALORI	1	1	0	1	1	4
PARTITO DEMOCRATICO	5	2	1	3	4	15
UNIONE DI CENTRO	1	1	0	1	0	3
VERDI	1	0	0	0	0	1
LISTA CIVICA MARCHE	0	0	0	0	0	0
IL POPOLO DELLA LIBERTA'	4	2	1	3	2	12
LEGA NORD	0	0	0	1	1	2
LADESTRA	0	0	0	0	0	0
INSIEME PER IL PRESIDENTE	0	0	0	0	1	1
RIFONDAZIONE COMUNISTI ITALIANI	0	0	1	0	0	1
SINISTRA ECOLOGIA LIBERTA'	1	0	0	0	0	1
Total votes	13	6	5	9	9	42

Table 9 – Measuring the deviation from fair shares for the 2010 Marche's Electoral result and for the solution of the MILP (target shares fair shares)

Index	2010 electoral result	MILP solution
max	1.36	0.93
sum	22.19	12.05
average	0.34	0.19

In view of the above discussion one can verify that, although the actual seat assignment produced by Marche's electoral procedure matches the district and list totals, it does not produce the "most" proportional solution. In fact, following to the best approximation principle, two alternative solutions produced by the MILP model are better than the 2010 electoral result whichever the index used to compare them. It must be noticed that, when measuring the deviation w.r.t. the exact Hare quotas (Table 7) the institutional seat assignment and the MILP solution perform quite similarly, while a considerable difference can be observed when deviation is computed w.r.t. the fair shares (Table 9). This result is probably due to the fact that, according to the Marche's law, the difference between each cell in the seat allocation matrix and the corresponding exact Hare quota are always less than or equal to 1, thus implying a small value of any global measure of deviation.

In Tables 6 and 8 we also highlighted the cells of the seat matrix of the alternative solutions provided by MILP differing from the corresponding ones in the 2010 electoral result. It can be noticed how the number of modified cells increases from Table 6 to Table 8, the reason still being related to the different target matrix considered by the MILP.

In the same spirit as Marche, Friuli Venezia Giulia issued its new electoral law in 2007¹⁸. As in other Regions, the number of seats at stake in each district is computed by applying the Hare quota and the Largest Remainders method, and there are provincial lists, groups of lists and coalitions.

The electoral system is a proportional one in which the distribution of seats is articulated at two different levels, taking into account an elaborated system of exclusion thresholds, governability and representation clauses (pursued through majority and minority prizes, respectively), and respect for ethnic minorities (Slovenian-speaking citizens). There is a single ballot and the voter may express a preference for a candidate Governor and one for a candidate of a provincial list; disjoint vote is allowed.

The candidate with the largest number of preferences is declared the winner and a majority prize guarantees that her/his coalition gets 55% or 60% of the total number of seats (in this computation, the seats for the Governor is included, too) depending on how many votes it actually received¹⁹. Similarly, groups supporting non-winner candidates Governor receive at least 40% of the total seats (this amount includes the seat reserved for the second classified candidate Governor). Thus, if such condition is not satisfied, a minority prize is applied²⁰. Within each coalition, the distribution of the seats is performed with d'Hondt method.

A specific feature of the case of Friuli Venezia Giulia is that within each constituency the quota is computed according to the Imperiali formula (i.e., the total number of valid votes is divided by the total number of seats plus 2 and then rounded down)²¹. A ranking of the lists is obtained in each constituency on the basis of their residual votes and it is used first for subtracting to lists seats in excess, and, then, for assigning the remaining seats to the lists of groups which did not receive all their seats up to this stage.

However, no check is performed to prevent the total number of seats allocated in a constituency exceeding the number of seats at stake in that constituency. In addition, by adopting the Imperiali quota the probability that lists of a given group receive in total more seats than those their group is entitled to increases, causing, in this case, more operations of subtraction of exceeding seats and new assignments of residual ones. Hence, in the case of Friuli Venezia Giulia an even higher degree of lack in double proportionality is produced.

¹⁸ Friuli-Venezia Giulia's Regional Law n. 06/18/ 2007.

¹⁹ Out of the 57 regional seats, the coalition supporting the winner candidate receives at least 60%, if the winner candidate casts more than 45% of the total votes received by all the candidates Governor, and at least 55% otherwise.

²⁰ An additional prize is reserved for the groups of lists formed by parties representing the Slovenian-speaking minority (minority group). For the sake of simplicity, details are omitted here.

²¹ Imperiali and Hagenback-Birkoff have the similar drawback in allowing the seats assigned to actually exceed the total number of seats (see footnote 10).

4. Conclusions

The history of electoral systems is full of examples of paradoxes. In the Italian case both at the national and regional level electoral laws generally aim to double proportionality at the territorial level on the basis of population and at the political level on the basis of votes received by lists. However, double proportionality is rarely achieved and, in the quest to find a balance between representation and governability, regional electoral laws have in most cases added complexity to the original seat assignment procedures without solving their intrinsic flaw. Indeed, there seems to be a persistent legacy of the bi-proportional “bug” in Italian electoral laws

Achieving double proportionality is not a simple task. With the current electoral rules it may occur that for some voting outcomes the system awards a territorial district more (or less) seats than those apportioned to it. Moreover, a specific device typically introduced in several regional electoral laws to ensure sufficient support to the Governor, produces further inflation of the already large regional councils.

The combination of different proportional formulae and mechanisms for handling residual seats do not necessarily satisfy the prefixed number of seats at stake in each district because they operate by considering one district at a time. This is not a negligible defect and it has serious practical consequences on which are the candidates actually elected, possibly triggering serious controversy.

The correction mechanisms introduced in some cases, such as in Marche, are not sufficient: although they satisfy the constraints of the problem they do not necessarily produce the “best” result in terms of proportionality. By extending to the matrix case the idea of finding a seat allocation which is the “closest” to a perfectly proportional (fractional) one, and using concepts such as the fair share, we were able to find two alternative solutions to Marche’s 2010 election which produce better results. Such results are obtained through the formulation of an optimization problem which is not any more complex than the current regional electoral formulae.

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